Mathematics Grade-Level Instructional Materials Evaluation Tool

Quality Review

GRADE 3

Council of the Great City Schools™

Textbooks and their digital counterparts are vital classroom tools but also a major expense, and it is worth taking time to find the best quality materials for students and teachers. While there is no perfect set of materials or textbooks, this Grade-Level Instructional Materials Evaluation Tool-Quality Review (GIMET-QR) is designed for use by professionals as a framework for evaluating the quality of instructional materials and choosing materials that are best suited to provide a coherent learning experience for students.

The district should begin its textbook adoption process by screening an entire publisher series with the Instructional Materials Evaluation Toolkit (IMET), developed by Student Achievement Partners, to see which ones are worthy of deeper consideration. The GIMET-QR can then be used to evaluate materials *for each individual grade*. But rather than providing an exhaustive list of grade-level standards, GIMET-QR starts with the progression to algebra continuum as the major area of focus, allowing for the in-depth review of a smaller set of mathematical concepts covered in the Common Core State Standards Mathematics (CCSS-M) at each grade level.

The GIMET-QR focuses on both the quality of the *content* and the instructional *design* of materials—with a specific focus on evaluating whether materials contain a balance of the three components of rigor (conceptual understanding, applications, and fluency) called for in CCSS-M. Unlike many tools that evaluate the presence or absence of required content, the GIMET-QR prompts reviewers to ask, "How *well* do the materials and assignments reflect and support the rigor of the CCSS-M?"

To answer this question, GIMET-QR contains Guiding Statements along with references to the CCSS for each statement. In response to each Guiding Statement, reviewers are asked to cite specific supporting evidence from the materials themselves, rather than relying on the table of contents or the topic headings. This supporting evidence can then be used to rate whether and to what degree the criteria have been met so that all students have access to a quality mathematics program.

It is important to keep in mind that quality is not defined as "compliance" or a mere checklist of topics. The GIMET-QR aims to help schools and districts choose materials that will provide the best overall learning experience for their students. The distinctive features of instructional materials, like style and appeal that contribute to engaging students in mathematics, should therefore be considered along with the mathematical content and cognitive demand.

The review process culminates with a summary in which reviewers cite strengths and weaknesses of the product, thus providing explicit details for the overall assessment. The summary may also indicate, prior to making a recommendation for purchase, any areas that district curriculum leaders may need to augment or supplement.

Please note: Acrobat Reader or Adobe Acrobat is required to complete this form electronically and save any data entered by users.

THE STRUCTURE OF GIMET-QR

The GIMET-QR for Mathematics is divided into four sections:

I. "CCSS-M" clusters and standards along the "progression to algebra continuum" for grade three

This first section focuses on the content of the materials under review and on the quality of the explanations and connections that develop the concepts and skills for the algebra continuum in grade three. This section features "guiding statements" that require reviewers to examine the quality of the materials, as well as the assignments that address the level of rigor in CCSS-M. The statements about materials and assignments are similar, but their focus is different. While the materials statements ask the reviewer to show evidence about the quality of how concepts and skills are attended to in the text or digital resource under review, the assignments statements ask the reviewer to cite evidence that students are given the opportunity to apply their understanding of those concepts and skills.

The statements in bold print in GIMET-QR refer to the CCSS-M clusters (i.e., 3.NBT.1-3) for reviewers to use in considering the quality of materials and assignments. The reviewer may notice that the wording of the cluster heading is somewhat different than what is written in CCSS-M. This was done to address what materials and assignments could offer in support of the cluster standards. However, the essential wording of the cluster headings is maintained. The standards indicated within GIMET-QR are listed as written in CCSS-M. In grade three, the "CCSS progression documents," from the Institute of Mathematics,[1] were used to provide additional specificity and clarity for the reviewers about what to look for in *Operations and Algebraic Thinking*, *Number and Operations – Fractions (Grades 3-5)*, and *K-5 Measurement*. This progression information within the document is indicated using an indentation and preceded by the symbol (▶).

II. Decision Recording Sheets: Quality Criteria for Conceptual Understanding, Applications, and Fluency with an accompanying rubric for high quality/exciting materials and assignments

The second section asks the reviewer to reflect on the findings from the first section to answer the question of how well the materials reflect and support the rigor of the CCSS-M. Reviewers are asked to consider how well the materials support teachers and engage students. Judgments are made after organizing the evidence around each of three dimensions of rigor—**conceptual understanding, applications,** and **fluency**. Reviewers assign one of three ratings: **High Quality/Exciting, Good Quality** or **Minimal Quality**. The section also includes a rubric which describes high quality/exciting materials and establishes the highest criteria for both materials and assignments.

III. Adoption Committee Recommendation Form

The third section, to be completed after reviewing multiple submissions for adoption, is an *Adoption Committee Recommendation Form*. This provides reviewers with an opportunity to list their top three choices and cite specific strengths and weaknesses for all of the materials being reviewed.

IV. Appendix

The fourth section is an Appendix that includes two items: *The Progression to Algebra Continuum* and a table of *Common Addition and Subtraction Situations*.[2]

GIMET-QR does not attend to all the grade three standards but rather only those listed within the progression to algebra continuum. GIMET-QR does not attend to coherence across grade levels but does look for coherence within a grade when considering the quality of materials and assignments. Similar to CCSS-M, GIMET-QR operates at a very fine grain size, while individual lessons and units under review might work across clusters. GIMET-QR is not a checklist that would fragment the CCSS-M, rather the "fine grain size" deliberately focuses on how well the materials reflect the intent of the CCSS-M.

1 University of Arizona Institute of Mathematics, http://ime.math.arizona.edu/progressions/

2 From pages 89-90 of the Common Core State Standards for Mathematics. Adapted from Box 2-4 of *Mathematics Learning in Early Childhood*, National Research Council (2009, pp. 32-33).

GETTING STARTED

Completing the GIMET-QR entails a five-step process. Reviewers are expected to read through each of the steps and their explanations, and locate all the pertinent tables and pages before starting. Then complete each step.

Step one – Individual reviewers will evaluate how well the materials and their accompanying assignments develop the algebra continuum content for each grade level. Use the tables that start on page four to capture the evidence of how and where the materials do this. The purpose for noting specific examples as evidence is to contribute to discussions with other reviewers in steps two through four. Cite specific examples of the explanations, diagrams, and pictorial representations in the materials and assignments that prompt students to show their understanding. Additionally, reviewers should consider the interaction of students with the materials in two areas: 1) students as receptive learners (interactions with the explanations and illustrations in the materials) and 2) students producing and showing their understanding (interacting and completing the assignments in the materials).

Step two – Discuss your findings and evidence with other reviewers. Reviewers should discuss the evidence cited and use it to confirm or assist you (individually) in reviewing and revising your findings.

Step three – Next, reviewers need to consider the interaction of students and teachers with the content of the materials along three dimensions of rigor—**conceptual understanding**, **applications**, and **fluency**—to assign a judgment of quality to each dimension. Reviewers should answer the question: How well do the materials reflect and support the rigor of the CCSS-Mathematics overall? Reviewers will use the guiding questions found in the **Decision Recording Sheet** together with the rubric describing **high quality** to assign ratings. Consider the totality of the collected evidence along the dimensions of rigor, and record your rating at the bottom of each table.

The highest level of quality is described using the words "**High Quality/ Exciting**." We use these words to indicate a high degree of excitement about the materials and the assignments. As the reviewer considers the descriptors, keep in mind that these criteria apply to each dimension of rigor for both the materials and the assignments they present to students. To earn this rating, the evidence must demonstrate grade-level rigor of the CCSS-M in an engaging way.

The other levels represent varying degrees of quality. For example, **"Good Quality"** indicates that the materials and assignments are workable or sufficient. **"Minimal Quality,"** meanwhile, indicates that the materials are sufficient on their own, but would not be conducive to motivating students.

These descriptions will be used for rating the overall quality of the program.

Step four – Discuss your findings and conclusions with other reviewers. Include the following questions as a part of the discussion:

- What are the top three strengths of the texts?
- What areas need improvement?
- What additional supports would be needed to implement the textbook series or digital materials?

Step five – After discussion, reach consensus and make final recommendations on the **Adoption Committee Recommendation Form.**

I. CCSS-M CLUSTERS AND STANDARDS

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.0A.1-4 Materials demonstrate and show how to represent and solve problems involving multiplication and division by: ■ Showing how to interpret products of whole numbers as equal groups or arrays, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each *(for example, describe a context in which a total number of objects can be expressed as 5 x 7)*. ▶ Including a variety of multiplication and division problems for each of the following: ***unknown product***, e.g., 3 x 8 = ☐; ***group size unknown***, e.g., If 18 inches of string are cut into three equal pieces, how long is each piece of string?; ***number of groups unknown***, e.g., If 18 pieces of candy are to be packed six to a bag, how many bags are needed?; showing how to determine the unknown number that makes the equation true in each of three equations: 8 x ☐ = 48; 5 = ☐ ÷ 3; 6 x 6 = ☐? ■ Illustrating whole-number quotients, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. ■ Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., using drawings and equations with a symbol for the unknown number to represent the problem. ▶ Illustrating the "equal groups" and "arrays, area" to lay the foundation for extending multiplication and division to algebraic expressions (for example, connecting ***unknown product with equal groups,*** e.g., There are three bags with six plums in each bag. How many plums in all?; ***equal group with group size unknown,*** e.g., If 18 plums are shared equally into three bags, how many plums will be in each bag?; ***arrays showing an unknown product, group size unknown,*** e.g., There are three rows of peaches with six in each row. How many peaches are there?). *continued on next page >*	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
continued from previous page > ■ Showing how to determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = ☐ ÷ 3, 6 x 6 = ?.* ▶ Attending to and developing the academic language students need to explain their reasoning about unknown products, group size unknown, number of groups unknown, and the relationship between all three. Students often have difficulty recognizing that each multiplication or division problem involves three quantities, each of which could be the unknown. Similarly, students must understand that in equal groups, the roles of the factors differ – which may present potential problems. One factor is the number of objects in a group (like any quantity in addition and subtraction situations), and the other is a multiplier that indicates the number of groups. So, for example, 4 groups of 3 objects is arranged differently than 3 groups of 4 objects. Thus, there are two kinds of division situations depending on which factor is the unknown (the number of objects in each group or the number of equal groups). Whereas in the array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. However, rows and columns do depend on the orientation of the array. "Row" and "column" language may be difficult for students, e.g., "The apples in the grocery window are in 3 rows and 6 columns," is difficult because of the distinction between the number of things in a row and the number of rows. There are 3 rows but the number of columns (6) tells how many are in each row. There are 6 columns, but the number of rows (3) tells how many are in each column. Students do need to be able to use and understand these words, but this understanding can grow over time while students also learn and use the language in the other multiplication and division situations. ▶ Laying the foundation for students to "see" the commutative property for multiplication in rectangular arrays and area through row and column language, e.g., when an array is rotated 90°, the rows become columns and the columns become rows. ▶ Focusing on the common structure across different problems.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/ MATERIALS
3.0A.1-4. Assignments ask students to represent and solve problems involving multiplication and division by: ■ Interpreting products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.* ▶ Solving a variety of multiplication and division problems for each of the following: ***unknown product***, e.g., 3 x 8 = ☐; ***group size unknown***, e.g., If 18 inches of string are cut into three equal pieces, how long is each piece of string?; ***number of groups unknown***, e.g., If 18 pieces of candy are to be packed six to a bag, how many bags are needed?; and determining the unknown number that makes the equation true in each of three equations: 8 x ☐ = 48, 5 = ☐ ÷ 3, 6 x 6 = ☐?. ■ Interpreting whole-number quotients, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7 or in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* ■ Using multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. ▶ Showing an understanding of "equal groups" and "arrays, area" by connecting multiplication and division. ■ Determining the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of three equations: 8 x ? = 48, 5 = ☐ ÷ 3, 6 x 6 = ?. ▶ Using academic language to reason about unknown products, group size unknown, and number of groups unknown; and describing the relationship between all three. ▶ Describing and illustrating connections between and across a variety of problem situations. ▶ Reading to understand the problem situation, representing the situation and its quantitative relationships with expressions and equations, and then manipulating that representation if necessary, using properties of operations and/or relationships between operations. ▶ Linking equations to concrete materials, drawings, and other representations of problem situations. (Note: These will help students develop an algebraic perspective many years before they will use formal algebraic symbols and methods).	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.OA.5-6. Materials show explicit connections between the properties of multiplication and the relationship between multiplication and division by: ■ Illustrating how properties of operations are used as strategies to multiply and divide. Examples: *if 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). Similarly, 3 x 5 x 2 can be found by 3 x 5 = 15 then 15 x 2 = 30 or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property).* ▶ Students need not use formal terms for these properties. ▶ Materials explain and exemplify the use of the properties of operations for multiplication and division to: 1) accomplish a purpose in a calculation, and 2) justify a step. ■ Providing illustrations of division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.	
3.OA.5-6. Assignments push students to model and apply the properties of multiplication and the relationship between multiplication and division by requiring them to: ■ Apply properties of operations as strategies to multiply and divide. Examples: *if 6 x 4 = 24 is known, then 4 x 6 = 24 is also known (Commutative property of multiplication). Similarly, if 3 x 5 x 2 can be found by 3 x 5 = 15 then 15 x 2 = 30 or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication). Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property).* Students need not use formal terms for these properties but are required to: ▶ Model, apply, and justify a calculation using the properties of operations for multiplication and division. ▶ Illustrate with drawings and equations how to apply the properties of operations as strategies to multiply and divide. ▶ Make the connection that two of the factors are quotients of the related division problems and that for every product there are two divisions. ▶ Relate the product, factors, or quotient to what it means in the context of a problem situation.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.0A.7. Materials develop and support students in fluently multiplying and dividing within 100 using strategies such as the relationship between multiplication and division by: ▪ Supporting the development of fluency (by the end of grade three, know from memory all products of two one-digit numbers). ▸ Illustrating and modeling decomposing and composing products that are known to find an unknown product, i.e., 7 x 5 can be found by finding 5 x (6 + 1); since 5 x 6 + 5 x 1 so 7 x 5 = 30 + 5 more which is 35. ▸ Organizing practice to focus on products that are understood but not yet known with reasonable speed and accuracy.	
3.0A.7. Assignments require that students fluently multiply and divide within 100 by: ▪ Applying strategies such as the relationship between multiplication and division (e.g., by knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations). By the end of grade three, know from memory all products of two one-digit numbers. ▸ Describing relationships within products by modeling decomposing and composing products that are known to find an unknown product, i.e., 7 x 5 can be found by finding 5 x (6 + 1); since 5 x 6 + 5 x 1 so 7 x 5 = 30 + 5 more which is 35. ▸ Explaining the relationship between area and multiplication and addition, representing the relationship in multiple ways (i.e., pictures, graphs, concrete materials, tables, etc.); and applying this to problems involving multiplication and area.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.0A.8-9. Materials show how to solve problems involving the four operations, and identify and explain patterns in arithmetic by: ■ Showing how to solve two-step word problems using the four operations; representing these problems using equations with a letter standing for the unknown quantity; and assessing the reasonableness of answers using mental computation and estimation strategies including rounding. ■ Showing and re-focusing attention on arithmetic patterns (including patterns in the addition table or multiplication table), and explaining them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*	
3.0A.8-9. Assignments require that students solve problems involving the four operations, and identify and explain patterns in arithmetic by: ■ Solving two-step word problems using the four operations, representing these problems using equations with a letter standing for the unknown quantity, and assessing the reasonableness of answers using mental computation and estimation strategies including rounding. ■ Describing and illustrating arithmetic patterns (including patterns in the addition table or multiplication table) and explaining them using properties of operations. For example, *observe that 4 times a number is always even, and explain why 4 times a numbers can be decomposed into two equal addends*. ▶ Focusing on products that are understood but not yet known with reasonable speed and accuracy. ▶ Requiring that students: 1) use extended reasoning and modeling as they apply the four operations in problem situations involving properties, measurement (length and area), and estimation of intervals of time, liquid volumes, or masses of objects, and 2) write explanations with embedded symbols, graphs, etc.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.NF.1-3. Materials develop and support students' understanding of fractions as numbers by: ■ Showing a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; showing a fraction *a/b* as the quantity formed by *a* parts of size *1/b*. ▶ In grades one and two, students used fraction language to describe partitions of shapes into equal shares. In grade three, students apply the idea of equal shares as they develop the idea of a fraction more formally, building on the idea of partitioning a whole into equal parts. The whole can be a shape such as a circle or rectangle, a line segment, or any one finite entity susceptible to subdivision and measurement. ▶ Grade three students start with a unit fraction formed by partitioning a whole into equal parts and taking one part, e.g., if a whole is partitioned into 4 equal parts then each part is ¼ of the whole, and 4 copies of that part make the whole. ▶ Students begin visualizing unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of whole numbers. Just as every whole number is obtained by combining a sufficient number of ones, every fraction is obtained by combining a sufficient number of unit fractions. ■ Showing a fraction as a number on the number line and representing fractions on a number line diagram. • Representing a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Showing that each part has size *1/b* and that the endpoint of the part based on 0 locates the number *1/b* on the number line. ▶ There are two important aspects that link to attending to precision (MP.6 - *Mathematical Practice 6*): specifying the whole and explaining what is meant by equal parts. • Representing a fraction *a/b* on a number line diagram by marking off *a* lengths *1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line. *continued on next page >*	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
continued from previous page > ▶ Materials develop and support student understanding that **fractions are numbers, unit fractions can be the measure of a length** (1/4 inch) just like a whole number (2 inches), **a fraction names a point on the number line**—just like a whole number does, and **fractions, like whole numbers, express the length from zero on the number line**. ▶ A common misconception for students is perceiving the unit on a number line diagram. When locating a fraction on a number line diagram, they might use as the unit the entire portion of the number line. *For example, on a number line marked from 0 to 4, they may indicate the number 3 when asked to find 3/4.* ▶ Materials alert teachers to common student misconceptions about fractions. ■ Showing equivalence of fractions in special cases, and comparing fractions by reasoning about their size. ● Understanding two fractions as equivalent (equal) if they are the same size, or the same point on a number line. ● Recognizing and generating simple equivalent fractions, e.g., *½ = 2/4, 4/6 = 2/3*). Explaining why the fractions are equivalent, e.g., by using a visual fraction model. ● Expressing whole numbers as fractions, and recognizing fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* ● Comparing two fractions with the same numerator or the same denominator by reasoning about their size. Recognizing that comparisons are valid only when the two fractions refer to the same whole. Recording the results of comparisons with the symbols >, =, or <, and justifying the conclusions, e.g., by using a visual fraction model.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.NF.1-3. Assignments require that students show and describe their understanding of fractions as numbers by: ■ Showing a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; showing a fraction *a/b* as the quantity formed by *a* parts of size *1/b*. ▶ The assignments prompt students to use and produce visual and concrete representations of unit fractions and equivalent fractions with particular emphasis on the number line. ▶ Students use appropriate academic language in describing partitions of shapes and build on the idea of partitioning a whole into equal parts. ▶ Students use various representations to illustrate connections between and among partitioning circles or rectangles, a line segment, or any one finite entity susceptible to subdivision and measurement. ▶ Students illustrate/show unit fractions as the basic building blocks of fractions, in the same sense that the number 1 is the basic building block of whole numbers; and just as every whole number is obtained by combining a sufficient number of ones, every fraction is obtained by combining a sufficient number of unit fractions. ■ Showing a fraction as a number on the number line and representing fractions on a number line diagram. • Representing a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Showing that each part has size *1/b* and that the endpoint of the part based on 0 locates the number *1/b* on the number line. • Representing a fraction *a/b* on a number line diagram by marking off *a* lengths *1/b* from 0. Recognize that the resulting interval has size *a/b* and that its endpoint locates the number *a/b* on the number line. ▶ Students use number line diagrams to show that **fractions are numbers, unit fractions can be the measure of a length** (1/4 inch) just like a whole number (2 inches), **a fraction names a point on the number line** just like a whole number does, and **fractions, like whole numbers, express the length from zero**. *continued on next page >*	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
continued from previous page > ■ Showing equivalence of fractions in special cases, and comparing fractions by reasoning about their size. • Understanding two fractions as equivalent (equal) if they are the same size, or the same point on a number line. • Recognizing and generating simple equivalent fractions, e.g., *½ = 2/4, 4/6 = 2/3*). Explaining why the fractions are equivalent, e.g., by using a visual fraction model. • Expressing whole numbers as fractions, and recognizing fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* • Comparing two fractions with the same numerator or the same denominator by reasoning about their size. Recognizing that comparisons are valid only when the two fractions refer to the same whole. Recording the results of comparisons with the symbols >, =, or <, and justifying the conclusions, e.g., by using a visual fraction model.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.MD.1-2. Materials show students how to solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects by: ▪ Showing how to tell and write time to the nearest minute and measure time intervals in minutes. Solving word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problems on a number line diagram. ▪ Showing how to measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Adding, subtracting, multiplying, or dividing to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using diagrams (such as a beaker with a measurement scale) to present the problem. This excludes multiplicative comparison problems (problems involving notions of "times as much").	
3.MD.1-2. Assignments require students to solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects by: ▪ Telling and writing time to the nearest minute and measuring time intervals in minutes. Solving word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problems on a number line diagram. ▪ Measuring and estimating liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Adding, subtracting, multiplying or dividing to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using diagrams (such as a beaker with a measurement scale) to present the problem. This excludes multiplicative comparison problems (problems involving notions of "times as much").	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.MD.5-7. Materials illustrate concepts of area and relate area to multiplication and addition by: ■ Showing area as an attribute of plane figures and illustrating concepts of area measurement. ● A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. ● A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. ▶ Materials should help students conceptualize area as the amount of two-dimensional space in a bounded region and to measure it by choosing a unit of area, often a square. ● Showing students how to measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). ■ Representing and connecting area to the operations of multiplication and addition. ● Showing how to find the area of a rectangle with whole-number side lengths by tiling it, and showing that the area is the same as would be found by multiplying the side lengths. ● Showing how to multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems. ● Showing how to use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. ● Showing area as additive by illustrating the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to show how to solve real-world problems. ▶ This includes showing students how to decompose (cutting and/or folding), re-compose, and eventually analyze with area-units by covering each with unit squares (tiles) and clearly distinguishing the attribute of area from other attributes, notably length. ▶ Developing the interpretation of the measurement of rectangular regions as a multiplication relationship of the number of square units in a row and the number of rows. This relies on the development of spatial structuring. To build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows, materials ask students to draw rectangular arrays of squares and learn to determine the number of squares in each row with increasingly sophisticated strategies, such as skip-counting the number in each row and eventually multiplying the number in each row by the number of rows.	

GUIDING STATEMENTS	SPECIFIC EVIDENCE FROM THE TEXT/MATERIALS
3.MD.5-7. Assignments ask students to illustrate concepts of area and relate area to multiplication and addition by: ■ Applying area as an attribute of plane figures and illustrating concepts of area measurement. • A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. • A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. ■ Measuring areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). ■ Representing and connecting area to the operations of multiplication and addition. ▶ The assignments push students to explain and connect area to multiplication and addition. Students represent this relationship in multiple ways (i.e., pictures, graphs, concrete materials, tables, etc.) and apply this to problem situations involving multiplication and area. • Finding the area of a rectangle with whole-number side lengths by tiling it, and showing that the area is the same as would be found by multiplying the side lengths. • Multiplying side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems. • Tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. • Illustrating area as additive by finding the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to show how to solve real world problems. ▶ Prompting students to determine the area of rectilinear figures in increasingly sophisticated ways by composing and decomposing them into non-overlapping areas and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.	

II. DECISION RECORDING SHEET

Completed by: ____ Date: ____________________________

*Use the evidence that you collected for grade three to begin judging the overall quality of the program. Begin by answering the overarching question: **How well do the materials reflect and support the rigor of the CCSS-M?** Use the accompanying rubric which describes the criteria for high quality/exciting materials and assignments that engage both students and teachers.*

Rigor requirement (balance): A program that emphasizes only fluency is not rigorous. Likewise, a program that only focuses on applications or conceptual understanding is not rigorous. For a program to be rigorous, there must be a balance of all three (conceptual understanding, applications, and fluency) as indicated in the grade level standards. By the end of grade three, there are specific fluency requirements for students (know from memory all products of two one-digit numbers and fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction), and standards addressing procedural skill (procedural skill refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing procedures flexibly, accurately, and efficiently).

Criteria for Rigor and Quality in Conceptual Understanding, Applications, and Fluency

CONCEPTUAL UNDERSTANDING: **CONNECTIONS**	
Materials: ■ How well do the materials develop conceptual understanding of operations and algebraic thinking as defined in the CCSS-M and in the *Progression to Algebra (Appendix A)*? ■ How well do the materials connect to and extend prior knowledge? • The materials present and describe explicit connections to prior knowledge, connections among mathematical ideas, and connections among different mathematical representations, using appropriate academic language. ■ How well do the materials develop academic language (including words, phrases, and sentences using symbols, graphs, and diagrams)?	**Assignments:** How well do the assignments prompt students to produce explanations and viable arguments? • The set of assignments challenge students to use their mathematical knowledge, academic language, and skills to solve problems and formulate mathematical models in a variety of contexts. ■ How well do the assignments ask students to make explicit connections to prior knowledge, connections among mathematical ideas, and connections among different mathematical representations?

CONNECTIONS: CRITERIA FOR MEETING THE RATING OF "**HIGH QUALITY/EXCITING**"		
	Materials *The materials present and describe explicit connections to prior knowledge, connections among mathematical ideas, and connections among different mathematical representations, using appropriate academic language.*	**Assignments** *The assignments in the materials encourage and challenge students to use their mathematical knowledge, academic language, and skills to solve problems and formulate mathematical models in a variety of contexts.*
Student	*Using high quality/exciting materials, my students will:* ■ comprehend the concepts and connections in the materials. ■ make sense of the mathematics. ■ be excited to try the problems and learn from working on them. ■ want to learn the mathematical concepts and gain confidence that effort to learn will pay off.	*Using high quality/exciting assignments, my students will:* ■ engage in the challenge of comprehension and discussion. ■ make sense of the mathematics. ■ be excited to try the problems and learn from working on them. ■ want to learn the mathematical concepts and gain confidence that their effort to learn will pay off.
Teacher	*Using high quality/exciting materials will help me:* ■ see and understand the mathematical goals of the lesson/unit. ■ understand better the mathematics that I am teaching, learn more mathematics from the materials, and want to learn more from interacting with students. ■ be excited about teaching the lessons and see how students respond to the connections in the lesson/unit. ■ focus students' efforts on the mathematical connections and give them feedback on how to do better. ■ anticipate typical misconceptions, missing connections, and which struggles will be most productive for students. ■ be confident students will be motivated to learn from and connect the mathematics, as well as gain confidence that their efforts to learn will pay off.	*Using high quality/exciting assignments will help me:* ■ want to learn more from interacting with students, analyzing their work on assignments, and re-engaging them in the concepts related to the assignments. ■ use students' responses to focus their efforts on the mathematical connections and give them feedback on how to do better. ■ anticipate typical misconceptions, missing connections, and which struggles will be most productive for students. ■ know students will be motivated to learn from and connect the mathematics as well as gain confidence that their efforts to learn will pay off.
RATING – Compared to the criteria listed above, the materials I have just reviewed would be considered: ☐ 3) High Quality/Exciting ☐ 2) Good Quality ☐ 1) Minimal Quality		

CONCEPTUAL UNDERSTANDING: **EXPLANATIONS**	
Materials: ■ How well do the materials provide example explanations connecting different representations to show why a statement or steps in an argument or solution is true and under what conditions it is true? ● The materials provide example explanations, using appropriate concepts and academic language for the grade level, to show how a way of thinking about a problem makes sense using several representations and explicitly identifying correspondences across representations. ■ How well do the materials use abstractions and generalizations to communicate the mathematical structure that organizes seemingly scattered individual events or results?	**Assignments:** How well do the assignments require that student provide explanations using appropriate content and grade-level academic language? ● The set of assignments requires students to use appropriate content and grade-level academic language to explain why reasons and justifications for steps in a solution or an argument are valid and how the mathematical structure represents generalizations about a problem situation (context) mathematically to their peers and the teacher. ■ How well do the assignments ask students to use the mathematical structure to organize individual, seemingly scattered statements or results to represent generalizations mathematically to their peers and the teacher?

EXPLANATIONS: CRITERIA FOR MEETING THE RATING OF "**HIGH QUALITY/EXCITING**"		
	Materials *The materials provide example explanations, using appropriate concepts and academic language for the grade level, to show how a way of thinking about a problem makes sense using several representations and explicitly identifying correspondences across representations.*	**Assignments** *The assignments require students to use appropriate grade-level concepts and academic language to explain why reasons and justifications for steps in a solution or an argument are valid and how the mathematical structure represents generalizations about a problem situation (context) mathematically to their peers and the teacher.*
Student	*Using high quality/exciting materials, my students will:* ■ comprehend the explanations presented in the materials. ■ make sense of the mathematics of the lesson/unit. ■ be excited to try the problems and learn from working on them. ■ want to learn the related mathematical concepts and gain confidence that their effort to learn will pay off.	*Using high quality/exciting materials, my students will:* ■ engage in the challenge of comprehension and explanation with their peers and with me. ■ make sense of the mathematics of the lesson/unit. ■ be excited to try the problems and learn from working on them. ■ want to learn the related mathematical concepts and gain confidence that their effort to learn will pay off.

4 University of Arizona Institute of Mathematics, *K-3 Categorical Data; Grades 2-5 Measurement Data*, http://ime.math.arizona.edu/progressions/

Teacher	*Using high quality/exciting materials will help me:* ■ see and understand the mathematical goals of the lesson/unit. ■ understand better the mathematics that I am teaching, learn more mathematics from the materials, and want to learn more from interacting with students. ■ be excited about teaching the lessons and see how students respond to the explanations in the lesson/unit. ■ focus students' efforts on the mathematical explanations and give them feedback on how to do better. ■ anticipate typical misconceptions, struggles that are most productive for students, and ways to help students to revise their explanation.	*Using high quality/exciting materials will help me:* ■ want to learn more from interacting with students, analyzing their work on assignments, and re-engaging them on the concepts related to the assignments. ■ use students' responses to focus their efforts on the mathematical connections and give them feedback on how to do better. ■ anticipate typical misconceptions, struggles that are most productive for students, and ways to help students revise their explanations. ■ know students will be motivated to learn from and connect the mathematics as well as gain confidence that their efforts to learn will pay off. ■ prompt students to make their mathematical explanations clear in a way that others can understand and critique them.

RATING – Compared to the criteria listed above, the materials I have just reviewed would be considered:

☐ 3) High Quality/Exciting ☐ 2) Good Quality ☐ 1) Minimal Quality

APPLICATIONS

Materials	Assignments
How well do the materials develop students' expertise in the application of concepts appropriate for this grade level? • The materials show how to use mathematics to analyze problem situations, appropriate for the grade level, and provide examples of deploying the Standards for Mathematical Practice to make sense of problems. ■ How well do the materials support students' understanding of how to analyze problem situations, showing how to use mathematics to help make sense of problems?	How well do the assignments develop the application of grade-level concepts? • The assignments prompt students to use mathematics and the Standards for Mathematical Practice to help them make sense of a variety of problems and formulate mathematical models of real-world phenomena appropriate for this grade level. ■ How well do the assignments support students' understanding of how to formulate mathematical models of real-world phenomena, including explaining assumptions and explaining why the model serves its purpose in a reasonable way?

APPLICATIONS: CRITERIA FOR MEETING THE RATING OF "**HIGH QUALITY/EXCITING**"		
	Materials *The materials show how to use mathematics to analyze problem situations appropriate for the grade level and provide examples of deploying the Standards for Mathematical Practice to make sense of problems.*	**Assignments** *The assignments prompt students to use mathematics and the mathematical practice standards to help them make sense of a variety of problems, appropriate for this grade level, by asking students to formulate mathematical models.*
Student	*Using high quality/exciting materials, my students will:* ▪ apply the concepts and connect them to each other and their different representations. ▪ make sense of the mathematics of the lesson/unit. ▪ be excited to try the problems and learn from working on them. ▪ understand how to formulate and model problem situations mathematically. ▪ gain confidence that their effort to learn will pay off.	*Using high quality/exciting assignments, my students will:* ▪ be challenged to use their mathematics to comprehend, analyze, and make sense of the problem situation. ▪ make sense of quantities and their relationship in the math problem. ▪ represent the problem concretely and pictorially and represent it as an equation and explain how the two representations relate to each other. ▪ identify important quantities in a practical situation and map their relationships using such tools as concrete models, diagrams, and equations. ▪ formulate and model problem situations mathematically. ▪ engage in discussions with their peers and the teacher to make sense of the problem and learn from them. ▪ be excited to try the problems and learn from working on them. ▪ gain confidence that their effort to learn will pay off.
Teacher	*Using high quality/exciting materials will help me:* ▪ see and understand the mathematical goal of the lesson/unit. ▪ understand better the mathematics that I am teaching, learn more mathematics from the materials, and want to learn more from interacting with students. ▪ be excited about teaching the lessons and see how students respond to the problems/tasks in the lesson/unit. ▪ be confident I can focus students' efforts on the mathematical tasks/problems and give them feedback on how to do better. ▪ anticipate typical misconceptions, missing connections, and which struggles will be most productive for students. ▪ be confident students will be motivated to learn.	*Using high quality/exciting assignments will help me:* ▪ prompt students to make their mathematical thinking clear in a way that others can understand and critique it. ▪ want to learn more from interacting with students, analyzing their work on problems/tasks, and re-engaging them on making use of concepts related to them. ▪ use the student's responses to focus their efforts on strategic thinking and give them feedback on generalizing to other related applications. ▪ anticipate typical misconceptions, missing strategies, and which productive struggles will be most beneficial for students. ▪ gain confidence that their efforts to learn will pay off.

RATING – Compared to the criteria listed above, the materials I have just reviewed would be considered:

☐ 3) High Quality/Exciting ☐ 2) Good Quality ☐ 1) Minimal Quality

FLUENCY	
Materials: ■ How well do the materials focus on developing critical procedural skills and fluency (*adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers)* by the end of grade three? • *Materials show how procedural skills and the standard for fluency for this grade level (*adding and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers*) work and provide consistent opportunities for students to practice using the algorithm or procedure.*	**Assignments:** ■ How well does the set of assignments focus on developing critical procedural skills and fluency? • *The set of assignments prompts students to develop and demonstrate fluency for adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers by the end of grade three.*

FLUENCY: CRITERIA FOR MEETING THE RATING OF "**HIGH QUALITY/EXCITING**"		
	Materials *Materials show how the standard for fluency, adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers, works and provide opportunities for students to practice using the algorithm, procedure or formula.*	**Assignments** *The set of assignments prompts students to develop and demonstrate fluency when adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers.*
Student	*Using high quality/exciting materials, my students will:* ■ have a variety of different ways to practice using an algorithm, procedure, or formula to develop fluency. ■ self-assess areas of weakness and strengths in *adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers* and receive feedback on which area(s) to improve.	*Using high quality/exciting assignments, my students will:* ■ build skills in *adding and subtracting within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction, and knowing from memory all products of two one-digit numbers* flexibly, accurately, efficiently, and appropriately. ■ gain confidence that their efforts to learn will pay off.

Teacher	*Using high quality/exciting materials will help me:* ▪ see and understand how the work on procedural fluency supports the mathematical goal of the lesson/unit. ▪ be confident that I can focus students' efforts on building fluency, help students understand and correct their mistakes. ▪ be confident students will be motivated to learn.	*Using high quality/exciting assignments will help me:* ▪ want to learn more from interacting with students. ▪ use students' responses to focus their efforts on building fluency and give them feedback on how to do better. ▪ see how to help students understand and correct their mistakes. ▪ be confident students will be motivated to learn.

RATING – Compared to the criteria listed above, the materials I have just reviewed would be considered:

☐ 3) High Quality/Exciting ☐ 2) Good Quality ☐ 1) Minimal Quality

III. ADOPTION COMMITTEE RECOMMENDATION FORM

Based on the substantial evidence collected, please rank all the grade three materials you reviewed in the order in which you would recommend them for adoption. The program or materials with your highest recommendation should be listed as number one below. Please provide any comments you deem pertinent. Include answers to the following questions based on the evidence cited in your materials review:

- **What are the top three strengths of this text?**
- **What areas need improvement?**
- **What additional supports would be needed to implement the textbook series or digital materials?**

RECOMMENDED		
PROGRAM NAME/EDITION:		COMMENTS:
1		
2		
3		

continued >

NOT RECOMMENDED		
PROGRAM NAME/EDITION:		COMMENTS:
1		
2		
3		

Completed by: ________________________________ Date: ____________________

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationships and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities*

From the K, Counting and Cardinality; K–5, Operations and Algebraic Thinking Progression p. 9

APPENDIX B: COMMON MULTIPLICATION AND DIVISION SITUATIONS[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

Source: http://www.corestandards.org/Math/Content/mathematics-glossary/Table-2/

1 The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

2 Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

3 The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.