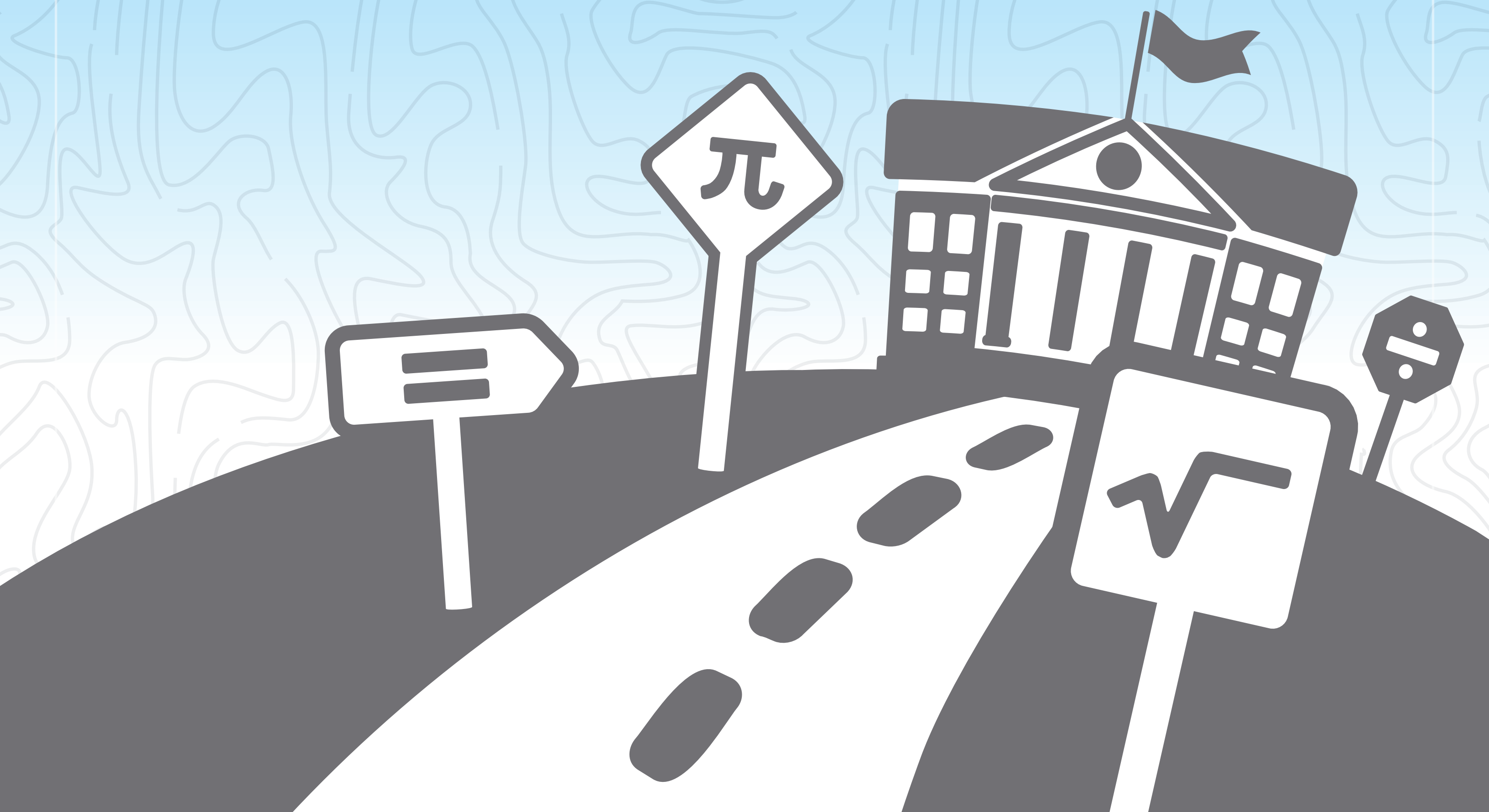
COUNCIL OF THE GREAT CITY SCHOOLS

1956 60TH 2016 ANNIVERSARY

A Framework for Re-envisioning Mathematics Instruction for English Language Learners

DECEMBER 2016



ABOUT THE COUNCIL

The Council of the Great City Schools is the only national organization exclusively representing the needs of urban public schools. Composed of 70 large city school districts, its mission is to promote the cause of urban schools and to advocate for inner-city students through legislation, research, instructional support, leadership, management, technical assistance, and media relations. The organization also provides a network for school districts sharing common problems to exchange information and to collectively address new challenges as they emerge in order to deliver the best education for urban youth.

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ACKNOWLEDGEMENTS

Schools in our Great Cities, and across the nation, are diversifying with increasing enrollment of English Language Learners—one of America's fastest growing student groups. The academic needs of these school children are complex and varied.

Fortunately, the achievement of these students is being taken seriously by urban educators across the nation. They have coalesced around a series of activities to ensure these children learn English and thrive in their studies of all subjects.

This document is one more piece of evidence of how urban school leaders are working to ensure success for all our students. It addresses two important needs. One, it provides a framework for understanding the interdependence of language and math in an era when the new college- and career-readiness standards in mathematics include unprecedented language demands. And two, it presents criteria by which school administrators and teachers can determine whether instructional materials being considered for implementation are well-suited for English Language Learners and are consistent with college and career ready standards for mathematics. Nothing like this has been previously attempted.

The intellectual horsepower that was involved in pulling this document together was impressive. An extraordinary team came together to discuss intensely complex and interconnected issues. I thank these extraordinarily talented and committed individuals, who include: Frances Esparza, Karla Estrada, Cathy Martin, Jennifer Yacoubian, Maria Crenshaw, Julio Moreno, Judy Elliott, Philip Daro, Harold Asturias, Lily Wong Fillmore, Judit Moschkovich, and Kevin Oh. Special thanks goes to Liz Gamino and our own Denise Walston who devoted numerous hours to pouring over the contributions of the experts and district practitioners of the team, and to the Council's ELL Team and Amanda Corcoran who brought this to completion. We also thank the school systems, universities, and organizations that permitted these individuals to work collaboratively on such an important initiative.

At this point, we hope that school officials and teachers across the country will use this document and the proposals and criteria in it to strengthen mathematics instruction for our English Language Learners and ensure they have materials that meet their needs.

Michael Casserly
Executive Director
Council of the Great City Schools

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SECTION I:
PREFACE

The Council of the Great City Schools is a membership organization of 70 of the nation's largest urban public school districts. These districts collectively enroll over a million English Language Learners (ELLs) or about 24 percent of the nation's total. The Council has a strong track record of initiating and working on policy, research, and programmatic efforts at the national and local levels to improve academic achievement among ELLs. Among other initiatives, the organization has produced groundbreaking reports and studies on how urban school systems improve the academic attainment of ELLs and comprehensive surveys on the status of ELLs in the nation's urban schools. In addition, the Council works directly with its member school districts to improve and support their instructional programs for ELLs through technical assistance, professional development, on-site reviews, meetings, and a national network of practitioners.

In conducting its work, the Council found that many urban school districts report significant difficulty finding high quality, rigorous, grade-level instructional materials that are written for ELLs at varying levels of English proficiency. This dearth of materials presents a substantial problem for urban districts that enroll sizable numbers of ELLs, and it is particularly acute at the secondary grade levels, where the complexity of content and text is higher than at the elementary grades. The adoption and implementation of new college- and career-readiness standards, as well as new state-level English Language Development (ELD) standards required by federal law, have only made this instructional need more obvious. This need was further documented by the Council's own field survey to gauge the perceived quality of instructional materials for ELLs. The results of this survey corroborated what has been common knowledge among urban educators for some time, i.e., quality instructional materials for ELLs are in short supply and the need has been exacerbated by the adoption of new standards.

In August of 2014, the Council released *A Framework for Raising Expectations and Instructional Rigor for English Language Learners*, a guide designed to define a new vision for English Language Development and to provide step-by-step guidance for selecting instructional materials, for English Language Arts, that will accelerate the acquisition of academic language and grade-level content for all English Learners in urban school districts. The Framework describes a re-envisioned English Language Development composed of two critical elements: Focused Language Study (FLS), and Discipline-Specific Academic Language Expansion (DALE). Language development and expansion (DALE) is expected to take place throughout the school day in all content areas of the curriculum. Because in a great majority of school systems, ELLs are more likely to be supported during their ELA instructional time than during mathematics or other content, the *Framework* included criteria for selecting materials that explicitly address the area of English Language Arts; it did not, however, address the area of mathematics. To articluate how DALE would take place within the context of mathematics, this companion document was developed to explicitly address the unprecedented role that language and communication play in service of understanding and applying mathematical concepts, under the new standards in mathematics. These new language demands in mathematics are particularly challenging for students who are learning English as a new language while they are also learning mathematical concepts.

Purpose and Audience

The overarching purpose of this document is to define a new vision for mathematics instruction that explicitly attends to the needs of ELLs, addressing the interdependence of language and mathematics. The following sections are devoted to (a) making clear that the grade-level college- and career-readiness mathematics standards are for ALL students, including ELLs, ELLs who require special education services, and any other students who face learning challenges in mathematics related to language needs; (b) articulating a theory of action in which ELL academic achievement improves when teachers provide all students with grade-level instruction, requiring a high caliber of materials and lessons that present high cognitive demand; (c) identifying and providing instructional principles and practices designed to address the language demands in the new standards for mathematics that may pose challenges for students who are developing both English proficiency and academic language in mathematics; and (d) providing criteria for the selection of instructional materials for mathematics that attend to academic language development and the language demands of the new standards for mathematical practices, so that ELLs and other students with language-related needs have access to grade-level content and practices as set by these standards.

Both the English Language Development and Mathematics Framework documents were developed to be applicable across grades K-12, and are designed to work in tandem with other tools that make grade-level distinctions for selecting instructional materials, such as the *Grade-level Instructional Materials Evaluation Tool-Quality Review* (GIMET-QR) and the *Instructional Materials Evaluation Tool* (IMET), or in combination with other evaluation protocols adopted by districts, as deemed appropriate by each district's instructional leadership.

Before selecting instructional materials for ELLs, however, districts must have a clear vision of how their instructional program for ELLs ensures attention to the instructional shifts and rigor of the college- and career-readiness standards, providing both the language development and the scaffolded grade-level content required for ELLs to be successful. To aid districts in this task, we have developed a framework for the interdependence of language and mathematics that is anchored in the language demands of the new standards and the needs for English language acquisition.

This document is designed for educators who are teaching mathematics to ELLs, whether in mainstream/general education classes, in self-contained classes for ELLs, or in other instructional contexts. It may also be used by teachers of students who have a high-incidence disability (e.g., Specific Learning Disability) and an Individualized Education Plan (IEP), or who have unfinished learning in mathematics due to language-related needs.[1] Though these constituencies are distinct, and each

1 This document does not address the particular needs of students with more severe disabilities, a low-incidence group that may not necessarily be receiving services in a general education classroom for most of their instructional time.

have unique needs, their needs may intersect when related to learning academic English and the interdependence between language and understanding complex mathematical content. No single method has proven effective in differentiating between English Learners who have difficulty acquiring language skills and those who have learning disabilities. As a result, schools, districts, and states struggle with the challenges of meeting the needs of these students. Throughout the document, we call out instances of specific considerations that our experts and practitioners have identified as being relevant for students from these distinct groups.

Finally, the document is meant to be a useful guide for anyone who is involved in the design, development, and/or selection of curriculum, materials, and resources, whether in a district's central office or at the school level. This includes administrators, principals, teachers (in general education and specialized areas), textbook evaluation committees, instructional leadership teams, resource teachers, math coaches, and content specialists.

Under IDEA 2004, Specific Learning Disability is defined as "a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations. The term includes such minimal brain dysfunction, dyslexia and developmental aphasia. The term does not include learning problems that are primarily the result of visual, hearing, or motor disabilities; of intellectual disability; of emotional disturbance; or of environmental, cultural, or economic disadvantage."

Additionally, under IDEA, a child may not be identified as a "child with a disability" primarily because he or she speaks a language other than English and does not speak or understand English well. A child may also not be identified as having a disability just because he or she has not had enough appropriate instruction in math or reading.

SECTION II:

RE-ENVISIONING MATHEMATICS INSTRUCTION FOR ENGLISH LANGUAGE LEARNERS

College- and career-readiness standards set new expectations for all students—including a deep understanding of core mathematical concepts, the ability to apply these concepts to real-world problems, and student participation in key mathematical practices, including fluency. In planning math instruction for a diverse array of learners, districts and states not only grapple with how to facilitate the development of conceptual understanding in mathematics, they must also address the specific needs of students who are simultaneously developing their English proficiency. As they respond to the required shifts within both the general education curriculum and ELL programming, districts need to ensure that their instructional practices and materials reflect a core set of foundational principles about academic language, teaching, and learning for mathematics.

Expectations and Agency

In recognizing the central role of *agency* and *authority* in student learning and progress, educators must embrace high expectations for ELLs and other students with language-related needs. Agency is defined as *the student's capacity and willingness to engage mathematically* and authority is defined as *the recognition for being mathematically capable*.[2] Both agency and authority are built through student's engagement in rigorous mathematical tasks and discussions that require them to conjecture, explain, construct mathematical arguments, and build on one another's ideas.

Yet many teachers are unsure of how to provide grade-level instruction when students are "so far behind," and may overuse the flexibility of resources to teach off level, which results in gaps of knowledge, concepts, and the language of mathematics. Changing this approach requires us to debunk the myth of fixed ability and build the fundamental expectation of access to the full content of the standards for ELLs and all students. As Jo Boaler mentions in *Mathematical Mindsets*, "Our education systems have been pervaded with the traditional notion that some students are not developmentally ready for some levels of mathematics… But these ideas are outdated, as students

2 Schoenfeld, A.H., Folden, R.E., & the Algebra Teaching Study and Mathematics Assessment Project. (2014). *An Introduction to the TRU Math Dimensions*. Berkeley, CA & E Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from http://ats.berkeley.edu/tools.html and/or http://map.mathshell.org/materials/pd.php

are as ready as the experiences they have had and if students are not ready, they can easily become so with the right experiences, high expectations from others, and a growth mindset."[3]

Teachers, therefore, need support in providing grade-level instruction and filling in "unfinished learning." Rather than aiming to equip students with only the learning necessary to perform each mathematical task or grasp each concept in isolation, teachers must also focus on instilling *horizon thinking* (an awareness of the larger mathematical landscape), on moving students to the next level, and on developing critical thinking through rigorous tasks and assignments. For example, the transition from arithmetic to algebraic thinking is a crucial horizon for teachers to consider. Teachers need materials that support this transition both conceptually and linguistically by helping students acquire skills such as developing the language to generalize about arithmetic situations.

Developing agency and authority, after all, requires creating opportunities for constructive engagement in mathematics and building on students' social and cultural knowledge and life experiences to develop not only conceptual understanding and related language competencies, but also the belief that mathematics is worthwhile, sensible, and feasible. And, in addition for students with learning disabilities (LD), a delicate balance in instruction should include maintaining cognitive rigor and sustaining persistent efforts to build capacity and proficiency in conceptual understanding and/or computational skills where weaknesses may be present.

The Interdependence of Language and Math

According to Judit Moschkovich, a professor and education researcher in the field of ELLs and mathematics, "Language is a socio-cultural-historical activity, not a thing that can either be mathematical or not, universal or not." She writes that "'the language of mathematics' does not mean a list of vocabulary or technical words with precise meanings, but the communicative competence necessary and sufficient for participation in mathematical discourse."[4]

Language in the math classroom, then, needs to expand beyond talk to consider the interaction of different systems involved in mathematical expression, i.e., natural language, mathematical symbols/systems, and visual displays. In recognition of this, teachers need to move away from a focus on simplified vocabulary and language toward a view that supports ELLs' productive engagement and participation in mathematical discussions. If we want students to use complex reasoning, engage in complex language, and participate in valued mathematical practices, teachers should focus less on a student's accuracy in using formal language as they learn English and more on students' mathematical reasoning and conceptual understanding, as reflected in their discourse practices.

3 Boaler, J. (2016). *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching* (pp. 8-9).

4 Moschkovich, J. (2012). *Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*. Retrieved from http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

The language of math can be particularly challenging for some students with language-based learning disabilities, resulting in confusion about terminology or difficulty following verbal explanations. In addition, for some students with learning disabilities, weak verbal skills affect their ability to monitor the steps of complex calculations. Instruction can therefore be most effective for these students if it allows for ample time to process verbal information that is 'chunked' into discrete segments. The 'chunking' of information is particularly important when asking questions, giving directions, presenting concepts, and offering explanations.[5]

Most students—but perhaps more so ELLs and students with learning disabilities—react to math word problems as a signal to do or solve something, rather than as meaningful sentences that need to be read for understanding.[6] It is therefore important to ask students to read or verbalize problems beforehand, and to verbalize their explanations of what they are doing as they solve a problem.

Language, in effect, should be understood as a complex meaning-making system,[7] and we may define mathematical discourse as communication that centers on making meaning of mathematical concepts.[8] While the language of mathematics is domain dependent (some of the language of geometry differs from the language needed to work with ratios and fractions, for instance), it nevertheless involves negotiating meanings by listening, responding, describing, understanding, making conjectures, presenting solutions, challenging the thinking of others, and connecting multiple representations, including mathematical notation and visual displays such as graphs, tables, and diagrams.

Attending to precision is one of the key mathematical practices delineated by many college- and career-readiness standards. This precision includes not only using precise words but, more importantly, making precise claims. Teachers need to model the practice of making precise claims and support students in using increasingly more precise ways of describing mathematical situations.

Finally, in considering the complex interaction between language and learning mathematics, students' everyday language and experiences should be understood and approached as resources, not as obstacles.[9] The home language of students and informal ways of talking are assets for reasoning mathematically and provide a springboard teachers can use to develop academic language and support mathematical understanding.

5 Garnett, Kate. "Math Learning Disabilities." *Division for Learning Disabilities Journal of CEC*, November 1998.

6 Ibid.

7 Moschkovich, J. (2012). *Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*. Retrieved from http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

8 Ramirez, N., & Celedon-Pattichis, S. (2012). Second Language Development and Implications for the Mathematics Classroom. In N. Ramirez & S. Celedon-Pattichis (Authors), *Beyond Good Teaching: Advancing Mathematics Education for ELLs*.

9 Moschkovich, J. (2012). *Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*. Retrieved from http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

Theory of Action: Re-envisioned Instruction for Developing Mathematical Language and Understanding

Given this core set of principles, what should effective mathematics instruction and materials for ELLs and other students with language-related needs look like, and how should they be experienced by students who require specific supports and differentiation related to language?

To begin, ELLs and other students with language-related needs *can* achieve college- and career-readiness standards in mathematics, engaging with complex mathematical concepts and solving real-world problems. If students are provided with productive opportunities to engage in rigorous mathematics instruction, high cognitive demand tasks, and discussions, they will build both understanding of complex mathematical concepts as well as procedural fluency.

Moreover, there is some evidence that the processes of developing language and developing conceptual mathematical understanding are interdependent and symbiotic. If ELLs and other students with language-related needs are encouraged and taught how to communicate their mathematical understanding and reasoning, their mathematical learning will serve to reinforce and advance their development of English proficiency.

Further, for students with learning disabilities, especially language deficits, it is important to develop the practice of reading or saying the mathematical problems before and after they solve them to understand that mathematics is not simply problems on a page, but rather meaningful sentences that need to be read for understanding.[10]

We also believe that teachers should use data to drive instruction, and should be given sufficient planning and instructional time by school and district leaders in order to attend to the thoughtful and strategic employment of language-focused supports in mathematics classrooms. District leaders should also ensure that teachers are provided with related, high-quality professional development and instructional materials that facilitate rigorous instruction aligned to grade-level college- and career-readiness standards. This will equip and empower teachers to ensure that ELLs and other students with language-related needs can engage meaningfully in complex, grade-level mathematics, and in the meaningful expression of their mathematical reasoning.

10 Garnett, Kate. "Math Learning Disabilities." *Division for Learning Disabilities Journal of CEC*, November 1998.

SECTION III:

KEY INSTRUCTIONAL PRINCIPLES AND PRACTICES

What, then, are the instructional principles for effectively building mathematical understanding and skills among students with language-related learning needs? What practices will further this vision for instruction?

To begin, we need to define what we mean by mathematical proficiency. Our working definition of proficiency involves five intertwined strands of knowledge and skills:[11]

1. Conceptual understanding, or comprehension of mathematical concepts, operations, and relations;
2. Procedural fluency, or skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
3. Strategic competence, or competence in formulating, representing, and solving mathematical problems (novel problems, not routine exercises);
4. Adaptive reasoning, or logical thought, reflection, explanation, and justification; and
5. Productive disposition, a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

We can think of procedural fluency as *knowing how to compute*. Although mathematical proficiency is often reduced to procedural fluency in arithmetic, this is only one component of a complete version of mathematical proficiency. For full mathematical proficiency, students need to learn more than computation or symbol manipulation. Conceptual understanding, strategic competence, and reasoning are as, if not more, important than fluent arithmetic computation—for example, understanding the applications of mathematics and knowing when to apply specific computations.[12]

Conceptual understanding is fundamentally about the relationships and *meanings* that learners construct for mathematical ideas, operations, solutions, or situations: knowing the meaning of a result (what a number, solution, or result represents), knowing why a procedure works, or explaining why a particular result is the right answer. Other aspects of conceptual understanding are connecting procedures to concepts and connecting procedures to multiple representations such as words,

11 Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding It Up: Helping Children Learn Mathematics*.
12 Hiebert, J., & Carpenter, T. P. (1992). Learning and Teaching with Understanding. In D. A. Grouws (Ed.), *Handbook of research in teaching and learning of mathematics* (pp. 65-97).

drawings, symbols, diagrams, tables, graphs, or equations.[13] Reasoning, logical thought, explanation, and justification are closely related to conceptual understanding. Student reasoning is evidence of conceptual understanding when a student explains why a particular result is the right answer or justifies a claim or conclusion. For example, when students make connections between multiplication and division, they recognize that multiplication can be conceived as repeated addition,[14] and can model multiplication using number lines, arrays, area models, and using base ten blocks. Moreover, students are able to make connections between and among these various representations.

It is crucial to note that conceptual understanding is *not* about teaching students to memorize formal and precise definitions of mathematical concepts. This typical misunderstanding of what conceptual understanding entails leads instruction right back to a focus on memorization. When teaching English Learners this is especially important to clarify, since a focus on precise use of words at the expense of mathematical reasoning can derail the development of conceptual understanding and mathematical proficiency in its fullest sense of the five strands of mathematical proficiency.

In an effort to advance this deeper, more nuanced definition of mathematical proficiency, today's college- and career-readiness standards call for several shifts from traditional mathematics instruction:[15]

1. **Balancing conceptual understanding and procedural fluency:** Instruction should (a) balance student activities that address both important conceptual and procedural knowledge related to a mathematical topic and (b) connect the two types of knowledge.

2. **Maintaining high cognitive demand:** Instruction should (a) use high cognitive demand math tasks and (b) maintain the rigor of mathematical tasks throughout lessons and units.

3. **Developing beliefs:** Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable.

4. **Engaging students in mathematical practices:** Instruction should provide opportunities for students to engage in a set of core mathematical practices:[16] (1) making sense of problems and persevering in solving them, (2) reasoning abstractly and quantitatively, (3) constructing viable arguments and critiquing the reasoning of others, (4) modeling with mathematics, (5) using appropriate tools strategically, (6) attending to precision, (7) looking for and making use of structure, and (8) looking for and expressing regularity in repeated reasoning.

13 Ibid.

14 In the early grades, repeated addition might be used to help students develop preliminary intuition about multiplication. However, repeated addition does not contribute to an understanding of multiplication required in higher grades, particularly when students progress beyond operations with natural or "counting" numbers, and move to using negative values, irrational numbers, etc.

15 Moschkovich, J. (2012). *Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core*. Retrieved from http://ell.stanford.edu/sites/default/files/pdf/academic-papers/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

16 Standards for Mathematical Practice. (2016). Retrieved from Common Core State Standards Initiative website: http://www.corestandards.org/Math/Practice/

But what will these instructional shifts look like in the classroom? To begin, tasks and assignments for ELLs and all math learners should be at a high level of cognitive demand, mathematically rigorous, on grade level, and make explicit connections between new and prior concepts. Teachers should ensure that ELLs have the opportunity to engage in *productive struggle,* allowing them sufficient time to make sense of a task or problem before intervening. ELLs—as well as all math learners—should be in classroom environments that make ample use of multiple modes of communication (speaking, listening, reading, writing), multiple representations (pictures, diagrams, tables, graphs, visual displays), and multiple communication settings (working with a peer, in small groups, making presentations, sharing written explanations, and critiquing the reasoning of others.) that allow students to express their mathematical reasoning, describe their solutions to problems, and develop understanding of mathematical concepts. Classroom instruction should also facilitate academic discussions focused on mathematical ideas and support exploratory and explanatory talk and writing. Finally, and only when necessary, teachers should strategically employ scaffolds specifically targeted to meet an individual student's educational needs or academic difficulties, while ensuring that this scaffolding does not compromise their access to rigorous mathematics content or their development of higher order conceptual understanding.

Each of these key areas of instructional practice is considered in detail in the following sections.

Employing Rigorous Tasks and Assignments

As with all students, the tasks and assignments used when working with ELLs must be rigorous, on grade level, and reflect high expectations. Classwork and assignments that students encounter should not be limited to memorizing facts, rules, or only carrying out calculations, but should extend to showing, describing, and discussing the underlying mathematical meaning of those procedures.

Instruction, therefore, should consistently employ cognitively demanding tasks that challenge students' mathematical thinking, problem-solving, and communication. Listening, speaking, reading, and writing about mathematics should not be approached as "enrichment" activities, but rather as integral parts of mathematics instruction to support students' understanding. Teachers should maintain this high level of rigor throughout lessons and units, supporting students as they progress in understanding mathematical concepts and allowing them to continuously develop and communicate their mathematical reasoning.

Like all students, ELLs also benefit from making explicit connections to mathematics learned before, so tasks and assignments should help students make connections among concepts and among various forms of mathematical representations.

It is important to recognize that, though some students may not have the language required to express their understanding of sophisticated mathematics concepts as a result of prior learning (or different methods of cognitive processing), they could be quite mathematically advanced. Some students may show the need for remedial math during elementary years when computational accuracy is heavily stressed but go on to join honors classes in higher math courses where conceptual understanding is more highly valued. For example, students in algebra may be able to explain when quadratic equations have complex solutions using conceptual understanding of the graph and the discriminant b^2-4ac

but have computational difficulty when applying the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ to solve equations. So, ELLs and other students with language-related needs may require multiple entry points, along with other appropriate supports for language development and communicating their understanding of mathematics, to allow for productive struggle while maintaining the high cognitive demand of the task.

To ensure a balance between access and mathematical rigor, teachers need to take a hard look at what students are asked to do. What are the tasks? Are they worthy of time and effort? Are students producing tables, graphs, and mathematical arguments (productive language) or are they just reading them? Through their tasks and assignments, teachers should ensure that they build experiences that are both receptive (learning and understanding the mathematics) and productive (doing, explaining, clarifying, connecting, and illustrating evolving understanding).[17]

So what does this look like in a classroom?

- Teachers design lessons and use resources that demand rigorous teaching and grade-appropriate learning, expecting students to employ higher-order thinking and communication skills such as explaining, conjecturing, and justifying.
- Teachers expect students to demonstrate deep conceptual understanding of mathematics—for example, through explanations of why a procedure works and how it connects to a previously-learned procedure or algorithm, the use of multiple representations to show the meaning of a procedure, or the successful completion of application-based assignments and tasks that require more than regurgitating facts or definitions and using basic procedural skills.
- Teachers support students in making sense of and solving problems rather than directly guiding them to answers. Teachers also ask students to justify their approaches and solutions to a problem. For example, when a teacher asks a student to explain how the/she solved a problem and the student says, "I divided," the teacher then asks, "Why did you divide?" or "What information in the problem led you to believe that division was the most efficient choice?".
- Teachers support students in demonstrating their understanding of procedures and their connections to underlying concepts, using academic language to communicate - either verbally or in writing - what they understand and how they reason or solve problems.
- When provided examples of and non-examples of a claim, concept, or strategy, students are able to analyze, verbally think out loud, describe the differences, and explain why these differences matter.

17 Baldinger, E., & Louie, N. *TRU Math conversation guide: A tool for teacher learning and growth*. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California Berkeley & College of Education, Michigan State University. Retrieved from http://ats.berkeley.edu/tools.html and or http://map.mathshell.org/materials/pd.php

Encouraging Productive Struggle

There is a pervasive myth that math is nothing more than learning how to produce answers. College- and career-readiness standards require an approach to mathematics instruction that emphasizes developing mathematical understanding, engaging in valued mathematical practices, and applying mathematical concepts to real-world situations. Making sense of mathematics inherently requires that students engage in productive struggle. Thus, there is a real need for students and teachers alike to acknowledge the value of productive struggle in developing mathematical understanding, even if this struggle may be amplified for some students who are simultaneously working toward English language proficiency, or who have particular language-related needs. Teachers should learn to distinguish between productive and unproductive struggle when solving math problems. "When students make a mistake while using a standard algorithm, obtain an incorrect answer, or have difficulty generating a strategy on a problem, it is easy to misdiagnose the error as indicating something more broadly about the level of a student's ability in mathematics."[18] Teachers should therefore resist the urge to lighten productive struggle, and instead, look for ways to retain the *productive* nature of the *struggle*. But what do we mean by *productive struggle*—and how should we define this along the two scales of language and mathematical content development?

Encouraging *productive struggle* does not mean grappling with difficulty for the sake of difficulty. Rather, educators must strike a balance between providing mathematical rigor and scaffolding and encouraging emergent thinking as students grow in their understanding of mathematical concepts. As students engage in productive struggle, teachers should create opportunities to go beyond simply asking for answers to asking students to explain their problem-solving approaches and reasoning. This includes supporting and critiquing the reasoning of others during classroom discussions. As students struggle to explain their mathematical thinking, teachers are granted a window into assessing their instructional needs.

Teachers should use common misconceptions as a driving force for learning more mathematics.[19] Teachers and students must recognize mistakes or misconceptions not as failure, but as opportunities for learning through productive struggle, focusing on making "errors a fruitful site for mathematical work."[20]

18 Battey, D. S., & Stark, M. (2009). Inequitable Classroom Practices: Diagnosing Misconceptions as Inability in Mathematics. In D. Y. White & J. S. Spitzer (Authors), *Mathematics for every student. Responding to diversity, grades pre-K-5*. National Council of Teachers of Mathematics.

19 Ibid.

20 Ball, D., & Bass, H. (n.d.). *With an Eye on the Mathematical Horizon: Knowing Mathematics for Teaching to Learners' Mathematical Futures*.

So what does this look like in a classroom?

- Teachers reinforce the habits of analyzing mistakes and persisting through problem solving struggles. They use examples and non-examples to guide student learning through error analysis.
- Teachers develop a classroom culture where students feel safe to take risks in solving problems and are unafraid to engage with mathematical challenges.
- Teachers provide tools to enhance students' ability to independently solve real-world problems.
- Teachers support student participation in core math practices set forth by college- and career-readiness standards (each math practice does not have to be the focus of every lesson, but students need to have opportunities to participate in all the math practices at some point).
- Teachers use appropriate scaffolding to allow students to think about the mathematics they are learning. It is not about "rescuing" students—it is about developing students' thinking rather than the teacher's thinking.
- Teachers balance "discovery" of knowledge and understanding through strategic student-led instructional activities focused on processes (with direct instruction when appropriate).
- Students justify their reasoning, communicate their reasoning to others, and respond to the arguments of others. This includes explaining the reasoning behind correct answers as well as the misconceptions behind incorrect responses, which enhances conceptual understanding of central math ideas.
- Students demonstrate agency, persistence, and independence in mastering mathematical content.

Employing Multiple Modes and Representations in Mathematics

The mathematics classroom should be as flexible as possible in terms of using language to support the development of conceptual understanding in mathematics. Classroom environments that make ample use of multiple modes of communication and representations in teacher presentations, written explanations, and classroom discussions help advance students' understanding of mathematics. Such environments provide students with the means to express the thinking behind their own reasoning and to discuss the meaning of another student's reasoning while referring to a public record on the board (math symbols, pictures, diagrams, text, etc.), instead of discussing only what another student said.

It is important to keep in mind that "multi-modal" and "multiple representations" means more than just listening, speaking, reading, and writing. For example, teachers' use of visual representations—such as gestures, drawings, mathematical symbols, models, and diagrams—can support mathematical thinking for ELLs and other students with language-related needs. A mathematical diagram or table organizing information in a word problem, when chosen carefully, can serve as an intermediate step between understanding the text of a word problem and representing a solution using math symbols, offering a visual anchor for talking about the mathematical structure, as well as any important linguistic features, of a word problem.

In the same way, Universal Design for Learning (UDL) removes barriers for learners and provides multiple ways for representing information, allowing action and expression to convey student learning and understanding. UDL impacts the why, what, and how of learning through multiple methods or opportunities for engagement, representation, action, and expression, respectively.[21]

Students' understanding deepens when they are given the opportunity to create and analyze diagrams, tables, and graphs to represent a problem concretely or pictorially, as well as verbally or in writing, and to make explicit connections between and among these various representations. Once the door of access and understanding is open, ELLs can further develop academic language and use it to engage in mathematical discourse.[22]

21 Universal Design for Learning Guidelines. (2014, November 12). Retrieved from National Center on Universal Design for Learning website: http://www.udlcenter.org/aboutudl/udlguidelines_theorypractice

22 Driscoll, M., Heck, D., & Malzahn, K. (2012). *Beyond Good Teaching: Knowledge for Teaching English Language Learners Mathematics: A Dilemma*. NCTM.

So what does this look like in a classroom?

- Teachers employ multiple modes of written and oral communication (including listening, speaking, reading, writing, or gestures), and multiple representations (including pictures, diagrams, tables, math symbols, objects or manipulatives, talk, and written text).
- Teachers provide varied opportunities to participate in the classroom using concrete tools, pictorial representations, computers, assistive and instructional technologies, and manipulatives.
- Teachers create multi-modal learning experiences for students to recognize patterns across multiple representations of mathematical ideas or procedures (for example, representing whole number multiplication not only with math symbols but also area models, arrays, number lines, and base ten blocks).
- Teachers promote students' use of alternative representations and solutions to problems, constructing diverse opportunities for repeated exposure to content.
- In addition to listening, speaking, reading, and writing about mathematics, students create, analyze, share, and discuss visual representations as they work through math problems.[23]
- Students use a variety of representations to communicate their thinking and make explicit connections between and among the representations.
- ELLs and students with language-related needs are actively engaged in learning and develop the confidence to communicate their mathematical understanding in different modes and representations, using both informal and more formal language.

23 Garnett, Kate. "Math Learning Disabilities." *Division for Learning Disabilities Journal of CEC*, November 1998.

Supporting Academic Language and Conversations

Mathematics instruction needs to support students in learning to reason mathematically and to express their mathematical reasoning. Precise mathematical language is highly valued by communities of people who use mathematics, such as mathematicians, scientists, and engineers. However, it is important to note that, for students learning mathematics, informal language is also important, especially when students are exploring a mathematical concept, learning a new concept, or discussing a math problem in small groups. This is called exploratory talk and can include informal language that reflects important student thinking. In other situations, such as when making a presentation or writing an account of a solution, more formal academic mathematical language becomes more important.

Such academic or formal mathematical language can be challenging for many students—especially ELLs. When we say *academic language*, we refer to language that falls into two categories: (1) technical, discipline-specific words and phrases used in the area of mathematics (such as hypotenuse, prime number, rational number, base-ten, "per," if and only if), and (2) all-purpose academic words—such as analyze, structure—that transcend the discipline of mathematics.[24] Mathematical discussions also involve much more than such language; they involve *discourse* practices such as going back to definitions, stating conjectures, making claims both explicit and precise, connecting claims to mathematical representations (such as a graph, table, or equation), generalizing across examples, and using counter examples. Moreover, when we talk about *academic mathematical discussions*, we refer not only to students sharing their solutions to a problem, but discussions where students are supported by the teacher in gradually developing more sophisticated language to articulate their mathematical reasoning, and to deepen their understanding and the understanding of other students through purposeful teacher or peer questions focused on the mathematics and the mathematical reasoning.

Teachers therefore need to carefully consider when informal ways of mathematically communicating are sufficient and when they are not, and how to support students in refining their informal language to gradually become more academic. While multi-modal representations of ideas support students as they use language while solving challenging mathematical tasks, educators must gradually support ELLs as they learn to express mathematical thinking and reasoning in more formal academic English and to engage productively in mathematical discussions with other students.

But how do you give teachers permission to stop and allow mathematical discussions to unfold? To begin, teachers should be encouraged to take time to highlight and clarify student strategies and mathematical thinking during whole class discussions, and to create opportunities for students to meaningfully interact by explaining, clarifying, justifying, and adding to the thinking of others. Moreover, teachers need to ensure the equity of all students' voices so that ELLs and other students with language-related needs feel empowered to participate and clarify their mathematical thinking in deep ways. These types of intentional math discussions facilitated by teachers help build experiences that are both receptive (students listening, watching, learning and understanding the

24 Driscoll, M., Heck, D., & Malzahn, K. (2012). *Beyond Good Teaching: Knowledge for Teaching English Language Learners Mathematics: A Dilemma*. NCTM.

mathematics) and productive (students doing, explaining, clarifying, connecting, representing, and illustrating their evolving understandings).[25]

So what does this look like in a classroom?

- Teachers model mathematical reasoning and academic language to support students as they learn to communicate the way they think through and solve mathematical problems.
- Teachers allow sufficient time for students to productively struggle with learning to communicate the thinking behind their solutions to mathematical problems. Teachers provide learning opportunities with appropriate scaffolds that encourage students to use more formal mathematical communication practices, including attention not only to precision in using words but also to whether students are making precise claims (for example, paying attention to constraints).
- Teachers establish classroom norms for how to ask purposeful questions of other students focused on mathematical reasoning and arguments and provide students with multiple opportunities to analyze their own and other students' solutions to problems.
- Teachers provide learning opportunities for using formal mathematics vocabulary *after* students have had direct experiences working on a math problem or concept, instead of pre-teaching vocabulary.
- Students encounter and solve mathematics problems expressed in a variety of formats (audio, text, etc.).
- Students are supported in refining their use of language to move towards more formal ways of describing, explaining, and justifying their reasoning in solving problems (both applied and not applied).

25 Baldinger, E., & Louie, N. *TRU Math conversation guide: A tool for teacher learning and growth.* Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California Berkeley & College of Education, Michigan State University. Retrieved from http://ats.berkeley.edu/tools.html and http://map.mathshell.org/materials/pd.php.

Using Strategic Scaffolding

The concept of scaffolding is often misunderstood and misinterpreted. Scaffolds should never entail a lower level of content, instructional rigor, or cognitive demand. Appropriate scaffolding provides an entry point for students to actively engage with cognitively demanding grade-level mathematics. It is not about "rescuing" students; instead, scaffolding empowers students to engage in, and ultimately emerge successfully from, productive struggle.

To ensure the appropriate and strategic use of scaffolds, teachers need to be explicit in the purpose of their use, when and why to use them, and when and how to remove them. Rather than suggesting generic strategies or one-size-fits-all scaffolds for ELLs and other students with language-related needs, scaffolds should be carefully selected and specifically targeted to reflect an understanding of students' previous experiences with mathematics instruction, their language development history, and their educational needs. For example, when identifying a student's educational needs or academic difficulties, it is essential to accurately determine whether the needs are indicators of developing levels of English proficiency, literacy gaps, a particular learning disability, or any combination of these factors.

Scaffolding should enable all students to be active participants in the mathematics classroom—reading, listening, discussing, explaining, writing, representing, and presenting—thereby not compromising student thinking, understanding, and communication.[26] Teachers must recognize when over-scaffolding impedes either the development of mathematical thinking or the language needed to express mathematical understanding and explain mathematical reasoning. And, teachers must gradually "fade" scaffolds, ensuring that students move to independence with complex, grade-level mathematical knowledge, skills, and conceptual understanding.[27]

26 *Application of Common Core State Standards for English Language Learners*. (n.d.). Retrieved August 18, 2016, from http://www.corestandards.org/assets/application-for-english-learners.pdf

27 Ball, D. L., & Forzani, F. M. (2011). Building a Common Core for Learning to Teach: And Connecting Professional Learning to Practice. *American Educator*, *35*(2), 17-21.

So what does this look like in a classroom?

- Teachers tap into their knowledge of students' needs to employ targeted scaffolding that develops their grade-level content knowledge, skills, reasoning, conceptual understanding, and language.
- Teachers are mindful of the pacing implications related to targeted scaffolding, and are supported by their administrators in taking time to select and use these scaffolds, ensuring that students with language-related needs can fully participate in grade-level mathematical work and practices.
- Teachers gradually "fade," or remove, supports, providing ample opportunities for students to independently demonstrate grade-level skills, content knowledge, reasoning, and conceptual understanding in mathematics. This allows students to develop agency, authority and identity.
- Teachers have access to school- and district-level professional development and resources so they can identify students' academic needs and select appropriate scaffolds.

SECTION IV:

CRITERIA FOR MATHEMATICS INSTRUCTIONAL MATERIALS

Effective instructional practices that provide access to grade-level mathematics and support the development of academic language in mathematics need to be supported by materials that are aligned to college- and career-readiness standards and designed to facilitate planning and delivery. In this section, we describe some general features that would indicate materials are appropriate for furthering grade-level mathematical understanding for ELLs.

To begin, a committee should be convened that incorporates members with multiple perspectives – including staff with expertise in mathematics instruction that is aligned with college- and career-readiness standards as well as those who understand the specialized needs of ELLs, students with disabilities, and gifted and talented students.

This tool is designed to help the members of this committee hone in on the specific features of materials that make them accessible and effective for English Language Learners and other students with unfinished learning in mathematics related to language needs and challenges, and may be used alongside such tools as the Grade-level Instructional Materials Evaluation Tool (GIMET), developed by the Council of the Great City Schools, as well as the tools developed by Student Achievement Partners (SAP).

The process of reviewing materials for their accessibility and alignment to college- and career-readiness mathematics standards entails three general levels of review:

- Level One: Overarching Considerations
- Level Two: Non-Negotiable Criteria and Considerations for ELLs
- Level Three: Additional Considerations

Overarching Considerations

The process of reviewing mathematics materials begins with an evaluation based upon general concerns, assumptions, and expectations that serve as a unifying foundation.

- ***Background knowledge, culture, and language as assets.*** Confirm that the materials recognize that students bring background knowledge to the classroom that can be used to advance their learning of mathematics. Specifically—

  a) Materials should explicitly state that all languages (including informal ways of talking and home languages) are assets, and that the home language and cultural practices of students are integral to their learning of mathematics.

  b) Assignments and learning experiences should value diverse backgrounds and empower students to effectively build upon their past learning experiences. Situations for applied problems used in materials should be as familiar and meaningful as possible for students, helping to bridge gaps between informal and formal learning experiences and inviting diverse learners to use their background knowledge to make sense of applied problems in instruction.

  c) Cultural contexts should respectfully reinforce and affirm students' multi-faceted identities by recognizing the assets of diversity rather than belittling identities with stereotypes and assumptions.

  d) Materials should equally emphasize various cultures and aspects of student identities and offer a wide range of views and perspectives, allowing all learners to meaningfully engage with the materials with the goal of developing students' academic identities as mathematics learners.

- ***Integrated attention to academic language development.*** Confirm that the materials explicitly address the language-related needs of ELLs who are learning mathematics in a new language (English), as well as the language-related needs of their English-speaking peers. In particular, ensure that:

  a) Materials are designed to address the interdependence of language, mathematical reasoning, understanding, and practices, supporting students' as they use and refine language to explain their mathematical reasoning and critique and build on the reasoning of others.

  b) Materials explicitly address the refinement from informal to formal mathematical language through activities and support that help students build on everyday informal language and move towards more formal academic mathematical language. This requires attention not only to discipline-specific terms (Tier III words such as angle, isosceles, etc.), but also to connection words and sentence structures that are particular to the language of mathematics (for example, given $x = 130$, solve for y or f(x)), as well as attention to typical math practices that are language intensive such as conjecturing, generalizing, making precise claims, and connecting claims to mathematical representations.

c) Materials provide tools to guide and structure mathematical discussions with a wide range of complex math texts (textbooks, word problems, assessment items, etc.), tasks, and expectations, as well as structured opportunities to revisit language over time with the goal of gradual development of the more formal language of mathematics.

d) Instructional materials support language development in all four modes (listening, speaking, reading, and writing).

- ***Standards alignment.*** Confirm an explicit and substantive alignment of materials to grade-level college- and career-readiness standards. In particular, assess whether:

  a) Instructional materials have passed a review using the Council's Grade-level Instructional Materials Evaluation Tool (GIMET).

  b) Materials support grade-level conceptual understanding in mathematics through rigorous tasks, high cognitive demand work, and applications (including applications to real life situations).

  c) Materials provide students with the opportunity to perform and apply a range of core mathematical practices.

  d) Materials make explicit connections to ELA college- and career-readiness expectations or "practices." For example, materials may connect to a specific genre of writing, such as journal writing, and how it could be relevant in mathematics classrooms.

Non-Negotiable Criteria and Considerations for ELLs

The Council has developed the following set of specific criteria related to language for selecting high quality, grade-appropriate mathematics materials that advance both conceptual understanding and language development for ELLs and other students with language-related needs.

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
Criterion I: The materials develop an in-depth understanding of key mathematical concepts.	
■ **Rigorous Tasks**	**1 2 3 4**
1. Materials cover complex conceptual ideas, with both examples and non-examples, addressing high-frequency misconceptions with clear explanations that provide strong language models for students.	
2. Materials attend to the development and expression of conceptual understanding where the grade-level standards set explicit expectations for understanding or interpreting. (IMET)	
3. Materials outline key mathematical concepts, essential questions, and corresponding standards.	
4. Materials include standards-aligned formative and summative assessments with rubrics, answer keys, and guidelines for scoring.	
5. Materials also include guidance for collecting and examining student work to assess conceptual understanding of key mathematical concepts, and to interpret student performance in accordance with various English proficiency levels.	
■ **Encouraging Productive Struggle**	**1 2 3 4**
6. Materials allow students sufficient time (before the teacher intervenes) to make connections to prior knowledge, connections between mathematical ideas and their different mathematical representations, and, when appropriate, in learning to use the relevant academic language. The progression of deeper mathematical understanding builds from one concept to the next.	
7. Materials include tools for students and teachers, including self-assessments and standards-aligned data trackers, to maintain a focus on developing and expressing deep understanding of concepts and student participation in mathematical discussions while also supporting fluency in mathematical computation.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Multiple Representations**	**1 2 3 4**
8. Materials reference and require students to make connections between linguistic and non-linguistic representations. This includes using a student's primary language, mathematical symbols, and using a variety of representations such as pictures, diagrams, drawings, graphs, tables, etc. For example, at the elementary level, materials may use pictures of 3D rectangular models to help students visualize "slicing" or decomposing the models into layers and smaller 3D rectangular models, or packing the 3D models with unit cubes to find the volume, and to relate the side lengths to the total volume through discussions, illustrations, and modeling.	
■ **Academic Language**	**1 2 3 4**
9. Materials provide experiences for students to participate in both receptive and productive language functions[1] while learning to use more complex, sophisticated, and precise language to express their mathematical ideas.	
■ **Strategic Scaffolding**	**1 2 3 4**
10. Materials define, illustrate, highlight, and encourage students to use both Tier II words that "cut across" all content areas (e.g., analyze, describe) and Tier III technical and discipline-specific words (e.g., hypotenuse, range, base-ten). Language and definitions, including those that are built through shared experiences in the classroom, must be accessible and usable to students, even if formulated in elementary terms.	
Criterion II: The materials ensure that students attain the fluencies and procedural skills required by grade-level college- and career-readiness standards.	
■ **Rigorous Tasks**	**1 2 3 4**
1. Materials provide a balance of important conceptual and procedural knowledge and connect the two types of knowledge.[2]	
2. Materials support student progress toward acquiring fluency in procedural skills— a. by developing students' conceptual understanding of the operations in question, b. by providing students with the mathematical language they need to communicate their increasing understanding, and c. by engaging students in meaningful and standards-aligned application tasks.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Encouraging Productive Struggle**	**1 2 3 4**
3. Materials provide sufficient time (before the teacher intervenes) for both exploration and repeated practice for developing procedural fluency throughout the year.	
4. Materials promote increasing independence in students' work with mathematical procedures based upon the grade-level fluency requirements, and provide opportunities for both exploratory and explanatory talk.	
■ **Multiple Representations**	**1 2 3 4**
5. Materials strategically use a variety of representations for students to make meaning of procedural skills as they engage in repeated practice. For example, materials use fraction strips or visuals of fraction bars to help students understand why dividing fractions involves reciprocals.	
■ **Academic Language**	**1 2 3 4**
6. Materials require students to communicate their mathematical reasoning about procedures using both informal and formal language to describe patterns and structure while developing procedural fluency.	
■ **Strategic Scaffolding**	**1 2 3 4**
7. Materials provide mathematical experiences that are both receptive (understanding the mathematical concept) and productive (doing, explaining, clarifying, connecting, and illustrating their evolving understanding of procedures).	
8. Materials provide supports for students' language development and use of academic language specific to mathematics.	
Criterion III: The materials allow teachers and students sufficient time to work with applications without losing focus on the major work of each grade.	
■ **Rigorous Tasks**	**1 2 3 4**
1. Materials include applications that are embedded in situations that are potentially familiar and/or meaningful to students and stress applying the major mathematics concepts of the grade.	
2. Materials include single-and multi-step application problems that develop the mathematical concepts (or ideas) of the grade, afford opportunities for practicing procedures, and engage students in solving problems.	
3. Materials include a balance of real-world problems and tasks that take students beyond only *memorizing and using procedures*. The complexity of tasks progresses to allow fundamental procedural skills, mathematical language, and conceptual understanding to develop across grades.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Encouraging Productive Struggle**	**1 2 3 4**
4. Materials allow students to spend sufficient time working by themselves (before the teacher intervenes) with application problems and tasks using appropriate scaffolds based on their language needs, without minimizing the complexity of the task.	
5. Materials support students in using mathematical ideas and engaging in mathematical practices to help them make sense of a variety of problems, develop mathematical models, and express their thinking.	
■ **Multiple Representations**	**1 2 3 4**
6. Materials facilitate students making sense of quantities expressed in different representations for solving problems.	
7. Materials reference and require students to make connections between linguistic and non-linguistic representations.	
■ **Academic Language**	**1 2 3 4**
8. Materials require that students communicate their mathematical reasoning while solving applied problems using either informal or formal language and attending to precision in calculations and claims.	
9. Materials provide teachers and students with purposeful and targeted activities for learning how to read typical mathematics texts. For example, materials provide opportunities and tools for extracting relevant information from word problems (such as highlighting, color-coding, and drawing attention to essential ideas) so that students learn to derive meaning from the text. Materials also encourage students to make connections between different types of word problems and real-world situations. Materials should also support students in learning to read textbooks, graphs, and tables used in applied problems.	
■ **Strategic Scaffolding**	**1 2 3 4**
10. Materials provide culturally-relevant examples of real-world applications for mathematical concepts in ways that motivate students to understand the content and spend time working with applications. For example, students may determine rates for how quickly news is disseminated through various print media compared to social media to understand measures of center and variability; or students may be asked to determine the estimated costs for heating and cooling their dream home in various climates, using a blueprint of the home to calculate area, volume, and surface area.	
11. Materials provide resources for students and teachers to bridge prior formal and informal mathematical knowledge to grade-level forms of mathematical reasoning and expression.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
Criterion IV: Materials provide students with the opportunity to develop and apply a core set of mathematical practices that enrich, rather than distract from, the major academic objectives of the grade.	
■ **Rigorous Tasks**	**1 2 3 4**
1. Materials support students in acquiring and refining language to *express* or *describe* how they: • make sense of problems, • use abstract and quantitative reasoning, • construct viable arguments and critique the reasoning of others, • make use of structure, and • see regularity in repeated reasoning. For example, in K-5, students look for regularity while learning addition and multiplication, the properties of operations, and the place value system, while in grades 6-8 students express regularity in repeated reasoning about proportional relationships and linear functions, or when they use regularity in mathematical operations to create equivalent algebraic expressions.	
2. Materials address the full spectrum of mathematical practices so that both assignments and tasks enrich and connect to the major work of the grade while highlighting the interdependence of language and mathematical understanding.	
■ **Encouraging Productive Struggle**	**1 2 3 4**
3. Materials describe the major work of the grade and each of the mathematical practices, including their language demands, for each lesson and unit. Materials also articulate how the mathematics in each lesson or unit reflects the major mathematical concepts of the grade.	
4. Materials encourage student engagement and participation in key mathematical practices. a. The key mathematical practices are reflected in assignments, activities, and problems that support and encourage students in developing the habits described in the practice standards. b. Assignments and tasks prompt students to: • take sufficient time (before the teacher intervenes) to make sense of problems and share strategies for solving problems, orally and in writing, • generate multiple approaches and representations, • explain and support viable arguments, and critique the reasoning of others, • and examine the validity of claims and solutions.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)			
■ **Multiple Representations**	**1**	**2**	**3**	**4**
5. Materials highlight opportunities for students to make connections between representations, generate and discuss multiple representations of mathematical concepts or procedures, communicate their thinking about multiple representations, and justify their reasoning while using multiple representations.				
6. Materials and assignments provide abundant and diverse opportunities for speaking, listening, reading, and writing, encouraging students to take risks, construct meaning, and seek reinterpretations of knowledge.				
■ **Academic Language**				
7. Materials model and support students as they develop both the language and the mathematical understanding to be able to participate in the full spectrum of mathematical practices requiring higher order thinking skills.				
8. Materials afford students the opportunity to actively use mathematical language to master the major work of the grade, focusing on students' mathematical reasoning, not on accuracy using language.				
■ **Strategic Scaffolding**	**1**	**2**	**3**	**4**
9. Materials provide examples of teacher-student actions and interactions that model and reflect the intent of the full spectrum of mathematical practices.				
10. Materials and assignments include robust problems with multiple entry points that display an arc of growing sophistication to support students' engagement in the full meaning of each practice standard as they refine their participation in the practice standards across grades and/or grade bands.				
Criterion V: Materials support the development of mathematical reasoning.				
■ **Rigorous Tasks**	**1**	**2**	**3**	**4**
1. Materials and assignments focus on reasoning (i.e., why a solution works, not only a description of the steps for a solution) with opportunities to examine, compare, analyze, and discuss examples of solutions to problems.				
2. Materials engage students in grade-level mathematical reasoning, deepening their understanding through speaking, listening, reading, and writing about their thinking and others' thinking.				
3. Materials encourage teachers to focus on ELLs' development of conceptual understanding rather than over-emphasizing precise use of language when not central to the task. They may, for example, draw attention to complex language constructs in mathematics, identifying errors which may be typical at different levels of English Language Proficiency (ELP), while helping to support ELLs in continuously expanding their command of the language of mathematics.				

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Encouraging Productive Struggle**	**1 2 3 4**
4. Materials allow students sufficient time (before the teacher intervenes) to construct viable arguments and critique the arguments of others using the grade-level mathematics ideas detailed in the content standards.	
5. Materials allow students sufficient time (before the teacher intervenes) to produce not only answers and solutions, but arguments, explanations, diagrams, and mathematical models, providing them with an opportunity to describe, analyze, and critique the reasoning behind solutions.	
6. Materials prompt teachers to prepare for a lesson by thinking about how to a) provide sufficient time before intervening, b) consider multiple student responses, approaches, questions, and possible misconceptions, and c) include opportunities for students to analyze and correct or address their own errors using mathematical reasoning.[3]	
■ **Multiple Representations**	**1 2 3 4**
7. Materials prompt teachers to prepare for a lesson by considering ahead of time how students might use multiple representations to describe, analyze, critique mathematical reasoning, and correct errors in problem-solving.	
8. Materials encourage students to relate multiple representations to academic language by requiring them to use multiple approaches and mathematical representations in solving problems and describing their reasoning.	
■ **Academic Language**	**1 2 3 4**
9. Materials include the specialized language of mathematical arguments, problem solving, and explanations. When necessary (i.e., for formal presentations, written work, etc.), that language is explicitly taught rather than assumed. Informal language used by students (especially in small groups) serves as a basis and resource for refining and introducing more formal language. For example, when students use the term “cancel,” materials should make a direct connection to the mathematical concept of equivalent expressions to avoid over generalization.	
10. Materials require students to use language in their explanations and arguments—even if it is informal or not perfect—to “piece” concepts together and build whole ideas in mathematics.	
11. Materials prompt students to transition between everyday informal language and formal mathematical language while employing multi-modal representations.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Strategic Scaffolding**	**1 2 3 4**
12. Materials support students in learning how to construct and critique arguments using both informal and formal textbook definitions and conceptual understanding to explain and justify their reasoning about mathematical ideas and solutions. (For example, materials describe how providing a counter-example is one way to construct an argument, but also highlight that examples alone do not establish a general statement).	
Criterion VI: The materials facilitate the use of a range of instructional approaches for a variety of learners.	
■ **Rigorous Tasks**	**1 2 3 4**
1. Materials provide students with opportunities to conjecture, explain, construct, and share mathematical arguments, as well as build on others' ideas, in ways that contribute to their development as *budding mathematicians*, confident in their ability to take on complex new mathematical challenges.[4]	
2. Materials consistently include extensions and/or more advanced tasks, activities, and lessons for students who are performing at or above grade level, supporting continuous language development for all learners. For example, elementary students who have developed proficiency with operations for "adding to" and "joining, separating, or comparing" (or "putting together") may work on more advanced problems, where they explore and apply the commutative and associative properties of addition.	
3. Materials consistently engage students who are performing below grade level in rigorous, content-related and standards-aligned tasks, activities, and lessons with targeted tools to progressively fill in unfinished learning, build skills, expand mathematical language, and increase independence.	
■ **Encouraging Productive Struggle**	**1 2 3 4**
4. Materials provide multiple opportunities and sufficient time (before the teacher intervenes) for students to detect and correct their own error patterns and to engage with grade-level content.	
5. Materials invite, support, and provide sufficient time for the active engagement of all students with the core mathematical ideas being addressed in a lesson.[5]	
6. Materials provide multiple entry points and explicit connections to prior knowledge that allow students to engage with lessons at their level of English proficiency in order to increase their depth of mathematical understanding.	
7. Materials allow sufficient time (before the teacher intervenes) for students to make meaningful connections between procedures, concepts, and applied problems presented in various ways (allowing for scaffolds and supports as appropriate).	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Multiple Representations**	**1 2 3 4**
8. Materials provide alternative ways to acquire new information, share mathematical reasoning, and participate in mathematical practices such as listening, reading, speaking, and writing in addition to engaging students in multiple modes of input (e.g., visual, kinesthetic).	
9. Materials use multi-modal representations to support development of academic language and mathematical concepts, and materials model for students how to use the various representations to communicate their knowledge.	
10. Materials require that students use multiple representations (talk, text, drawings, diagrams, math symbols, graphs, tables, etc.) as an intermediate step between the text (for example, a word problem or a textbook passage) and the symbolic (math symbols such as numbers, operations, or variables) phases of solving a mathematical task.	
■ **Academic Language**	**1 2 3 4**
11. Materials identify linguistic demands and offer appropriate instructional approaches, assignments, and tasks to support language development (English and, when possible, the home language), perhaps including a section on mathematical language.	
12. Language development is carefully considered while maintaining mathematical rigor. This includes— a. supporting students in making sense of the language of word problems without oversimplifying the text, b. paying close attention to the connections among a student's home language, mathematical symbols, and the use of multiple representations, c. highlighting cognates between mathematical terms in English that are shared with other languages, and d. providing activities and problems that lend themselves to instructional strategies such as "3 READS"[6] for word problems and other texts or graphic organizers that attend to the language of word problems and engage students with high-level language functions such as synthesizing, comparing and contrasting, and evaluating.	
13. Materials include tools that aid in the analysis and understanding of the language used for instructions, procedural exercises, and word problems to make sense of problems that are text based or language intensive.	
14. Materials engage students in activities that support both receptive and productive language functions (see ELPD for details) and experiences, meeting the demands of grade-level standards by providing meaningful supports.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Strategic Scaffolding**	**1 2 3 4**
15. Materials note instructional approaches suggested for whole class and differentiated lessons and activities.	
16. Materials include resources, if possible, that provide access to materials in students' native language. For example, native language might be used to preview or review concepts.	
Additional Criteria for Teacher's Edition	
■ **Rigorous Tasks**	**1 2 3 4**
1. Materials provide lesson overviews with rigorous standards-aligned and grade level content learning objectives, essential questions, standards-alignments, and sample agendas.	
2. Materials support teachers in planning effective, rigorous, and standards-aligned lessons for diverse learners with planning templates, sample instructional plans, and digital planning tools. Lessons should not be scripted to provide districts and teachers flexibility in planning for their curricular needs.	
3. Materials provide information about intentional math talk,[7] naming specific talk moves (e.g., talk to whole class, explain to a shoulder partner, follow, repeat, agree, disagree, comment) and using talk moves that focus on mathematical ideas, reasoning, understanding, and practices.[8]	
4. Materials provide opportunities for high level applications-based problems, activities, and projects with resources for facilitation, including background information, graphic organizers, worksheets, exemplars, and rubrics.	
5. Materials provide incorrect solutions based on common errors or misconceptions for students to analyze and compare to correct solutions with explanations of misconceptions leading to the incorrect solutions.	
■ **Encouraging Productive Struggle**	**1 2 3 4**
1. Materials provide information and examples of teacher moves to support mathematical discussions and students' explaining their reasoning.[9]	
2. Materials support teachers in establishing a classroom environment where students respect each other, learn to value each other's ideas, and learn to discuss the reasoning of others. Material's pacing guides and estimated time requirements for all activities are realistic in fostering such a classroom environment.	
3. Materials support identifying and building of multiple and frequent opportunities in lessons and units to pay attention to problem solving, reasoning, connecting multiple representations, and engaging in the eight math practices.	

Language-Related Criteria	Rating Scale (1-4, 4 being the best)			
4. Materials outline common errors and misconceptions for different mathematical topics and provides support for recognizing and remediating these misconceptions.				
5. Provides opportunities and guidelines for using embedded formative and summative assessments including tools for developing standards-aligned assessments (test bank and test maker) with answer keys and/or rubrics.				
■ **Multiple Representations**	**1**	**2**	**3**	**4**
1. Materials provide teachers with samples of different ways of student thinking at different grade levels and for different ways of expressing mathematical understanding at different levels of English proficiency.				
2. Materials provide teachers with samples of different ways of processing mathematical information in multiple modes.				
3. Rather than highlighting one representation and solution for problems, materials provide alternative representations and solutions.				
4. Materials suggest a variety of multi-modal resources and activities for teaching each topic with recommendations for implementation with learners at different levels of language development.				
5. Materials support technology integration with high quality interactive resources, including videos, presentations, and online features.				
■ **Academic Language**	**1**	**2**	**3**	**4**
1. Materials provide teachers with resources and models including word/phrase lists and concept maps to sustain academic vocabulary development with ELLs (words and phrases) in the context of mathematical work, to develop understanding of words referring to thinking and communicating.				
2. Materials provide teachers with resources and models for supporting students in developing language practices beyond vocabulary by focusing on the function (not the form) of mathematical claims and arguments.				
3. Materials provide content-related and standards-aligned informational texts to help engage students in content, make connections to real-world situations, and sustain language development that moves students along the English acquisition progression.				
4. Materials include content and language development grading guidelines and grading tools for students and teachers, which offer clear and helpful feedback concerning learning progress.				
5. Materials provide "can do" and "look for" indicators for students at various language levels with guidelines for supporting these students with language development along the English proficiency progression.				
6. Materials identify cognates and language teaching strategies to support content instruction.				

Language-Related Criteria	Rating Scale (1-4, 4 being the best)
■ **Strategic Scaffolding**	**1 2 3 4**
1. Materials provide look-fors, cues, etc. for teachers to examine student work to detect the evidence to determine what a student with particular high needs understands and needs in mathematics instruction.	
2. Materials provide explicit guidance about opportunities and strategies for re-engagement with mathematics when misconceptions and/or incomplete understandings occur.	
3. Materials provide specific guidance for flexible grouping and team facilitation/ management strategies as appropriate for team-based activities. These guidelines should be developed to enhance learning in specific activities rather than being a general listing of strategies.	
4. Materials provide specific differentiation recommendations with guidance on implementation for students with various needs tailored toward instructional strategies used in a particular lesson. These recommendations should not be generic "cover-all" strategies.	
5. Materials provide asset-based learning inventories to help facilitate flexible grouping and differentiation based on student strengths.	
6. Materials provide self-paced learning center activities, both paper and online-based, to build foundational skills and strengthen existing skills.	

Criteria Matrix References

1 See ELPD Framework, developed by CCSSO for details. Framework for English Language Proficiency Development Standards corresponding to th*e Common Core State Standards and the Next Generation Science Standards. (2012, September). Retrieved from Council of Chief State School Officers we*bsite: http://www.ccsso.org/Documents/2012/ELPD Framework Booklet-Final for web.pdf

2 Principles for Mathematic*s Instruction for ELLs. (n.d.). Retrieved from ht*tp://ell.stanford.edu/sites/default/files/math_learn-more_files/2.Principles%20for%20Math%20Instruction%208-14-13.pdf

3 For examples, see Smith, M. S., & Stein, M. K. (2011). Five Practices for Orchestrating Productive Mathematics Disc*ussions. NCTM. or Math Pathway*s & Pitfalls. (n.d.). Retrieved from https://mpp.wested.org/.

4 Schoenfeld, A. (2016). The Teaching for R*obust Understanding (TRU) Framework. Retrieved from* http://map.mathshell.org/trumath/TRU_framework_overview.pdf

5 Ibid.

6 SFUSD Mathematics Department. (2015, June). SFUSD Signature S*trategy #2: Three Read Protocol. Retrieved from ht*tp://www.sfusdmath.org/uploads/2/4/0/9/24098802/3_read_protocol_from_math_teaching_toolkit_2015-2016.pdf

7 Hintz, A., & Kazemi, E. (2014). Talking about Math. Ed*ucational Leadership,* 72(3), 36-40.

8 For examples, see: Chapin, S. H., O'Connor, C., & Anderson, N. C. (2009). Classroom Discussions: Using Math Talk t*o Help Students Learn, Grades K-6. Math Solutions.; Anderson, N. (2011). Classroom Discussions: Seeing Math Discourse in Action, Grades K-6: A Multimedia Professional Learning Resource. Houghton Mifflin Harcourt.; and Kazemi, E., & Hintz, A. (2014). Intentional Talk: How to Structure and Lead Productive Mathematical Discussions.*

9 For examples, see "Six Important Talk Moves" described in Resnick, L., O'Connor, C., & Michaels, S. (2007). Cl*assroom Discourse, Mathematical Rigor, and Student Reasoning: An Accountable Talk Literature Review.*

Additional Considerations

In addition to assessing the quality and appropriateness of instructional materials for ELLs, schools and school districts have to consider a number of additional factors that are critical to ensuring that English Learners and other students with specialized language-related learning needs receive high quality mathematics instruction. These factors include assessments, professional development, instructional technology, and interventions. Below, we provide a number of high-level considerations school district need to address in each of these areas.

Assessments

Assessments are integral to the learning process, measuring as well as facilitating student progress. As such, assessments must be designed to accurately and dependably provide information about student learning in order to guide instruction. The review committee should consider embedded formative assessments that meet the following criteria:

- The language of assessments should mirror the language of instruction, just as the tasks of assessment should mirror the tasks students have encountered in the classroom and in their assignments.[28]
- The contexts used in assessments need to be culturally-relevant in order to remove linguistic and cultural bias.
- Assessments, whether oral or written, should provide multiple opportunities for students to demonstrate rigorous, standards-based mathematics learning, reasoning, understanding, and practices in various ways and consistently throughout the learning process.
- Scoring guidelines and keys should support teachers in providing meaningful feedback to students about their progress and determining next steps.
- Student self-assessment opportunities should be provided to help students gauge their progress and increase their agency for continuous growth.

Professional Development

One of the leading challenges to ensuring rigorous, standards-aligned instruction for ELLs and students with language-based learning disabilities is the misconception that unfinished learning is an insurmountable obstacle to attaining grade-level proficiency. Many teachers will tell you they can't teach on grade level because their kids are "so far behind." Professional development, therefore, needs to be well targeted and provide ongoing job-embedded coaching to help teachers support grade-level instruction while filling in "unfinished learning."

28 Moschkovich, J. (n.d.). *Understanding Language: Principles for Mathematics Instruction for ELLs*. Retrieved from http://ell.stanford.edu/sites/default/files/math_learnmore_files/2.Principles%20for%20Math%20Instruction%208-14-13.pdf

In particular, professional development opportunities need to—

- Provide ongoing professional learning around how to engage ELL students and other students with language-related needs with visual representations and mathematical thinking tools, while regularly integrating language access and language production strategies into mathematics lessons.
- Bring together ESL and math teachers, as well as Special Education teachers, and provide guidance for collaboratively analyzing student work and recognizing student mathematical thinking about specific mathematical concepts.
- Provide coherent and systemic support throughout the organization to ensure that principals and administrators are supportive of new instructional practices in math. In particular, principals and administrators need to develop an understanding that language development and mathematical discussions are productive parts of the process of learning math.
- Highlight high-leverage research-based strategies for supporting and enhancing mathematical reasoning among ELLs.

In general, teachers should be provided opportunities to diagnose and assess their own professional learning needs and access differentiated learning as well as ongoing professional learning networks and communities through which best practices and resources are shared. Accessible on-demand resources including teaching videos, implementation guides, and toolkits designed to help diagnose and address common instructional challenges would be welcomed resources.

Strategic Use of Instructional Technology

Instructional technology has the potential to increase student engagement and deepen student understanding of mathematical reasoning. The review committee should look for resources that—

- Include scaffolds for ELLs and any other students with language-related needs and challenges that deepen understanding.
- Assist students in making connections among multiple representations (verbal, symbolic, abstract, visual, algebraic, etc.) and support students in expressing their reasoning using multiple representations.
- Guide teachers in using technology to support the development of mathematical reasoning and encourage student agency and independence in the learning process through student-paced instructional activities focused on building conceptual understanding.
- Support alternative research-based teaching models—flipped classrooms, blended learning, etc.—with digitally accessible instructional activities and resources.
- Support differentiated instruction among diverse learners in ways that provide opportunities for remediation, intervention, and enrichment through enhancing and expanding classroom content through online instructional resources.

- Support integration of content and activities onto Learning Management Systems (LMS)—such as Google Classroom, Edmodo, Canvas, Blackboard, etc.—with downloadable videos and assignment files.

Intervention

Finally, intervention materials should be selected to support specific diagnosed needs. It is assumed that intervention occurs after students have first had access to and opportunities for quality math instruction with differentiated support, and students demonstrate that they require additional intervention and focused instruction. Intervention strategies and materials will therefore vary according to purpose, age, and grade level. In general, the committee should ensure that—

- Any formal intervention programs are developed or purchased to augment current curriculum and are not considered a "replacement." They are intended to support and provide the learning students need to be successful in core instruction.
- Intervention strategies, support, and programs are designed to fill in student learning gaps, are directly connected to grade-level mathematical content, and include opportunities for students to develop conceptual understanding and participate in key math practices.
- Interventions do not leave students working only with lower grade-level work, or procedural fluency, and should not involve going back and re-teaching everything before students can proceed.
- Intervention is not only used for remediation. Interventions should also accelerate/ramp up students' knowledge from where they are, filling in gaps in understanding, and ensuring students are successful in core instruction.
- Interventions are data driven and designed with guidance on monitoring students' rate of growth on targeted areas, thereby limiting their use over time, and progressively encouraging students to gain independence.
- The purpose and outcomes of interventions are clearly defined and progress is monitored with data.

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